Control of Multi Tank System and Their Performance analysis

A THESIS SUBMITTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

In

Electronics & Instrumentation

by

ASHISH KUMAR SINGH

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Department of Electronics & Communication Engineering

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CERTIFICATE

This is to certify that the thesis entitled, "**Control of Multi Tank System and Their Performance analysis**" submitted by ASHISH KUMAR SINGH in partial fulfilment of the requirements for the award of Master of Technology degree in **Electronics and Communication Engineering** with specialization in "**Electronics & Instrumentation**" during session 2012-2014 at National Institute of Technology, Rourkela and is an authentic work by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any Degree or Diploma.

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Dedicated to my Respected Parents and Faculties and Beloved Friends

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Abstract

A multi tank level control system has an example of interacting and non-interacting system. In this system, we have considered three tanks each having equal cross section area and each tank can be assumed as a first order system which are connected in interacting and various non-interacting mode. The control system is intended to maintain the level of the third tank at some predefined value irrespective of changes of inflow of first tank. Conventional PID controller is a powerful controller used in process industries to regulate and control process variables. In this work, we also consider the effect of the disturbance on the response of the system. According to these disturbances, we need to implement feed forward controller with better tuning algorithm. Thus, we implement transfer function of above system and behavior is observed with step input.

Control of the level in the tank and flow in between them is basic problem in process control industries almost all the chemical industries are connected in cascade for storage of liquid and for other chemical processes. all the tank have their own manipulated variable to control liquid level inside the tank so in this section we are analyze the response of various type of tank, like conical, rectangular etc. the response of these tank which are connected in interacting and non interacting mode are observed with applying step input and response these system is improved by designing various type of controller like feedback controller and feed-forward controller. Effect of interaction is also observed and the effect of this interaction is minimized by designing a de-coupler circuit.

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CHAPTER: 1

INTRODUCTION

1.1 Background

Multi tank systems are widely used in chemical and petroleum industries so as to control the liquid level in the tank. It is the challenging task as it may affect both pressure and flow of the process so it is important to maintain level at set point. In this project the transfer function of the three tank system has been formulated. Three tank system is shown below

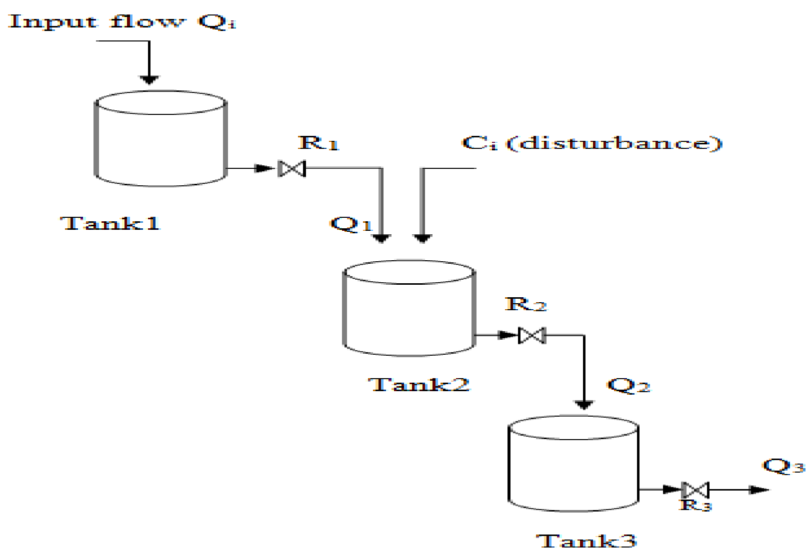

Fig. 1.1 Three tank Non-Interacting system

Various type of feedback controller can be used to control the liquid level in the tank but biggest problem is when disturbance come in to the picture. To nullify the effect of the disturbance we have to implement feed forward controller. To maintain liquid level at desired set point combination of feedback and feed forward-feedback controller is used.

1.2 Literature Review

Control of liquid level in any process control is challenging task. There are many different connection of tanks possible in the plant like Interacting and Non-Interacting. Many type of disturbances are possible which can affect the performance of the system. designing of controller for these type of process is challenging task in process control industries.

In paper [1]V.R.RAVI, proposed idea of Decentralized PID controller for interacting non linear system in which they are designed decoupling circuit for Two input two output process.

In paper [2] V.R.RAVI, proposed idea of adaptive control technique for non linear system in which they are represented as piecewise linear regions and for each linearized region, they tuned the PI controller.

In paper [3] V.R.RAVI, proposed idea of gain scheduling adaptive model predictive controller for two tank interacting system in which they are designed multiple linear MPC controller.

In paper [4] Parag, proposed the idea of decoupling design for two tank interacting system In which they are considered as two tank system as two input and two out process and then relative gain array is calculated and then corresponding pairing is done.

CHAPTER: 2

CONTROLLER DESIGN AND TUNING

2.1 Feedback control system

There are mainly two types of control loop existed in the industry namely negative feedback control and positive feedback control. In the positive feedback control the input and output values are added. In a negative feedback control the input and output values are subtracted. Generally negative feedback control systems are more stable than positive feedback control systems. By using Negative feedback the effect of random variations in values of component and inputs is minimized. There are many controllers which is used in industry i.e P, P+I, and P+I+D.[8]

Block diagram of feedback control system is shown in figure 2.1

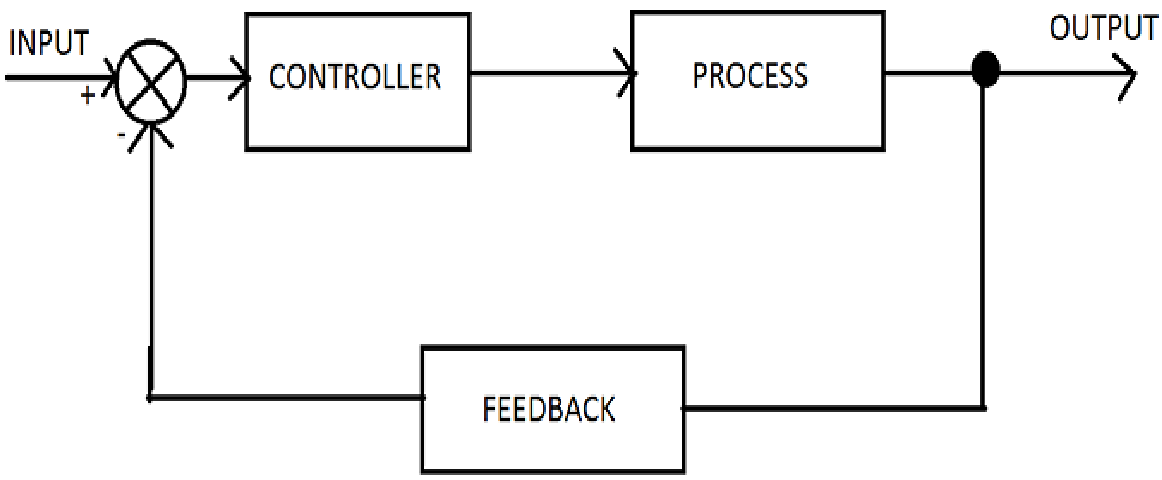

Fig.2.1 Block diagram of feedback control system

2.2 Feed forward control system

Feed forward controllers are used whenever there are major disturbances in the plant if we used feed-forward and feedback controller in combination the effect of disturbances at output of plant is minimized significantly. In ideal situation feed-forward controllers are capable to reduce the effect of measured disturbance to zero at output of process.

Feed-forward controllers are always used along with feedback controller feedback controller are used to tracking the change in set point and also to minimized the effect of disturbances which is unmeasured in the nature and such type of disturbances are always present in the real plant.[5-6]

Block diagram of feed forward control system is shown in fig. 2.2

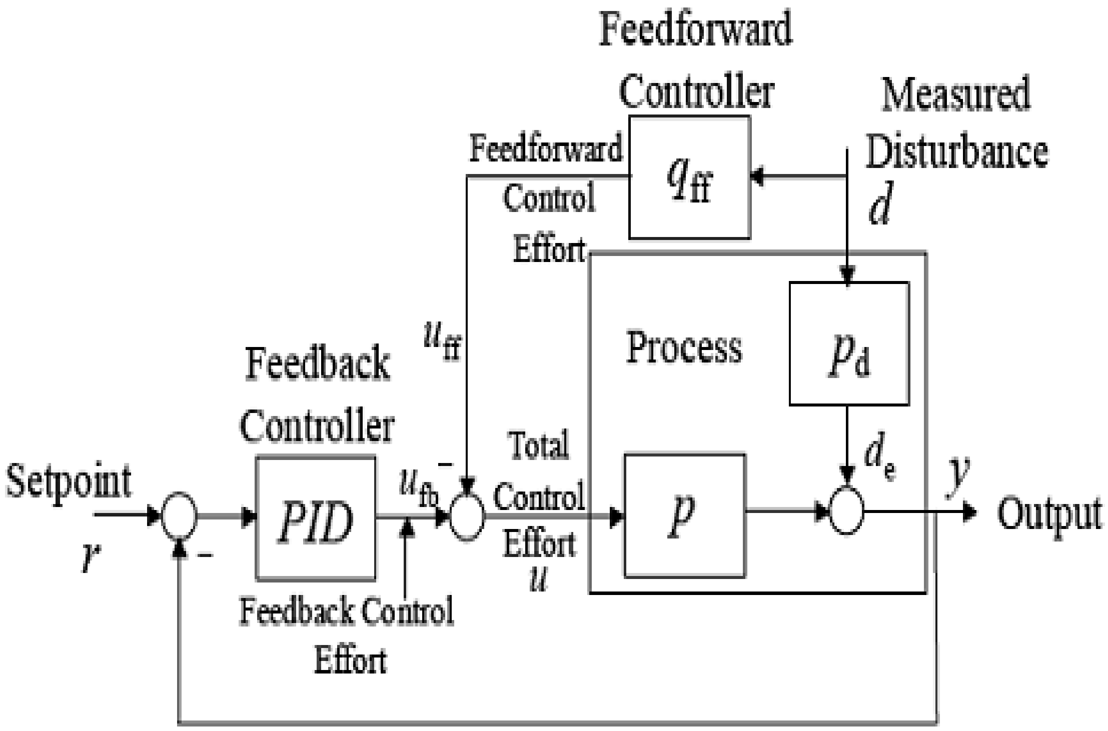

Fig.2.2 Block diagram of Feed forward controller

2.3 Tuning of PID controller

PID controllers are most widely used in process control industries so we have to tune PID controller in order to obtain desired response these methods are discussed below

2.3.1 Ziegler – Nichols Tuning Method

How to calculate the coefficient of P,I and D if the dynamic model of system is known and the dynamic model of system in not known[8-9].

2.3.1.1 If the dynamic model of system is not Known

If the dynamic model of system is not known then we have to test open loop response of the system experimentally by making feedback loop open and apply a step input to the process if the response is the s- shaped then Ziegler Nichols tuning method is applicable to the such type of the process otherwise it is not applicable the s type curve have two constant namely dead time and time constant of system these values are calculated by simply drawing a tangent.

This type response mathematically modelled as first order system having time constant τ and delay L

And then coefficient of P,I and D are calculated as follows

$$K_p = 1.2 \times \frac{T}{L} \tag{2.1}$$

$$\tau_i = 2L \tag{2.2}$$

$$\tau_d = 0.5L \tag{2.3}$$

2.3.1.2 If dynamic model of process is known

If dynamic model of process is known then to calculate the coefficient of P,I and D using the Ziegler Nichols tuning method by connecting only proportional gain in closed loop and vary the gain of controller till the oscillation is not observed in the output .

Gain at which oscillation is observed at output is called as critical gain of the system and using this critical gain we can calculate critical time period or ultimate time period and then PID controller can be tune by following formulae

$$K_p = 0.6K_{cr} \tag{2.4}$$

$$\tau_i = 0.5T_{cr} \tag{2.5}$$

$$\tau_d = 0.125T_{cr} \tag{2.6}$$

2.4 Tuning of Feed Forward Controller

In practical cases of feed forward controller we do not have neither block diagram of process nor transfer function of plant for these situation we can still tune the feed forward controller by making a step change as disturbance and then applying tuning rule for such type of system

To describe these rules we are assuming that its transfer function is in lead lag form

$$G_f = K_f \text{X} \left[\frac{(\tau_1 S + 1)}{\tau_2 S + 1} \right] \tag{2.7}$$

Where,

K_f - DC gain of the controller

τ_1, τ_2 - are the time constant of the controller

2.5 Block diagram of feed forward controller

Block diagram of feed forward controller is shown in fig.2.3 in which disturbance is added through the path (G_1)and effect of this disturbance is minimized by using feed forward controller (G_f).[7]

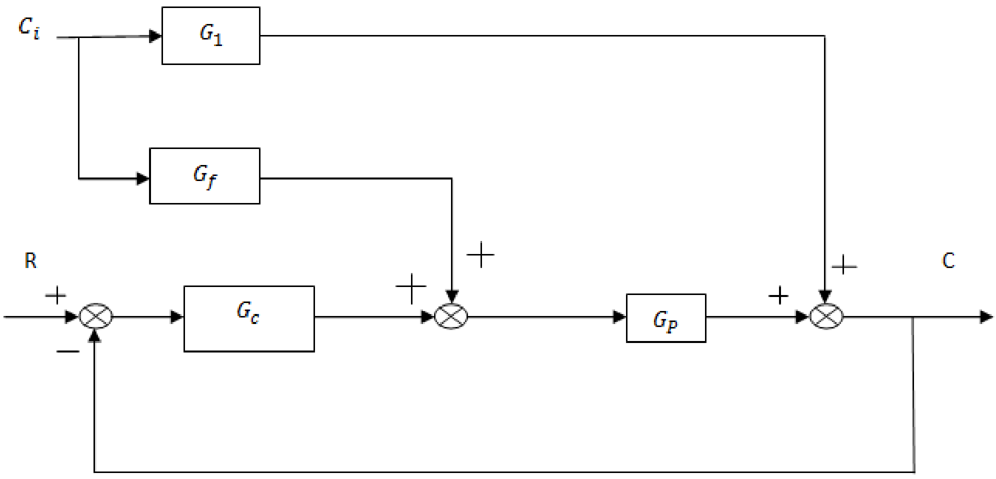

Fig.2.3 Feed forward- feed back control system

Where,

G_1 - transfer function between output and disturbance

G_f - transfer function of feed forward controller

G_c - transfer function of PI controller

G_p - transfer function of plant

2.5.1 Calculation of coefficient of feed forward controller

First make fee back path open and then step change in the disturbance input and observe the transient behaviour of output which may be lead dominant or lag dominant by identifying whether it is lead dominant or lag dominant we can calculate the value of τ_1, τ_2 by using formulae given in the table

Table 2.1 Tuning rule for Feed-forward controller.

Sr.No	Mode	τ_1	τ_2
1.	Lead	1.5 peak time	0.7 peak time
2.	Lag	0.7 peak time	1.5 peak time

CHAPTER: 3

Multi Variable Control

3.1 Multivariable Control System

If the process have only one output may be controlled by single manipulated variable these type of process classified as single input single output system while in process control industries each process requires more than one control variables and hence minimum two loops are connected with it. Any system having more than one control loops are known as multi input multi output or multivariable system.

There are mainly two types of loop interaction namely p-canonical and v- canonical it is clear from diagram p-interaction represent feed forward control while v- interaction represent feedback control

P- canonical representations

On the loop basis input and output can be related as following equation

$$y_1 = \left(u_1 * g_{11} + u_2 * g_{12}\right) \tag{3.1}$$

$$y_2 = \left(u_1 * g_{21} + u_2 * g_{22}\right) \tag{3.2}$$

Where,

y_1, y_2 - is the output of the system

u_1, u_2 - is the manipulated variable

The above relationship can be represented in more generalized in matrix form

$$Y = g * u \tag{3.3}$$

V- canonical representations-

Mathematical model for v- canonical representation

$$y_1 = \left(y_2 * g_{12} + u_1\right) * g_{11} \tag{3.4}$$

$$y_2 = \left(y_1 * g_{21} + u_2\right) * g_{22} \tag{3.5}$$

In matrix form it can be written by

$$y = \left(I - G_m G_i\right) * G_m U \tag{3.6}$$

3.2 Relative gain array

Relative gain array is not only important tool for pairing manipulated variable and control variable but also used to predict the response of the system. for (2×2) relative gain matrix are constructed as follows:

Let us consider the K_{ij} is the gain of transfer function G_{ij} assume manipulated variable u_2 is constant and if we make step change in manipulated variable u_1 of magnitude Δu_1 it will produce a change of Δy_1 in the y_1

So the gain between y_1 and u_1 when manipulated variable u_2 is kept constant

K_{11} at constant $u_2 = \dfrac{\Delta y_1}{\Delta u_1}$ at constant u_2

In place of manipulated variable u_2 if we consider the y_2 is constant then step change in manipulated variable u_1 of magnitude Δu_1 it will produce another change in y_1.

Gain in this condition is written by

K_{11} at constant $y_2 = \dfrac{\Delta y_1}{\Delta u_1}$ at constant y_2

Relative gain

$$\lambda_{22} = \frac{\left[{\Delta y_2}/{\Delta u_2} \right]_{u_1}}{\left[{\Delta y_2}/{\Delta u_2} \right]_{y_1}} \tag{3.7}$$

λ_{11} is dimension less, and it is ratio of output y_1 to input u_1 and it gives following information.

3.2.1. If λ_{11} = 0 then chance in the manipulated variable u_2 does not influence output y_1 and hence it should not be used for control of y_1.

3.2.1. If λ_{11}=1 it means the $\left(K_{11}\right)_{u_2}$ and $\left(K_{11}\right)_{y_2}$ have the same value therefore gain between output y_2 and input u_1 does not affected by the loop between output y_2 and input u_2

For a (2×2) there are three more relative gain i.e

$$\lambda_{12}=\frac{\left[\Delta y_1/\Delta u_2\right]_{u_1}}{\left[\Delta y_1/\Delta u_2\right]_{y_2}} \tag{3.8}$$

$$\lambda_{21}=\frac{\left(K_{21}\right)_{u_2}}{\left(K_{21}\right)_{y_1}} \tag{3.9}$$

$$\lambda_{21}=\frac{\left[\Delta y_2/\Delta u_1\right]_{u_2}}{\left[\Delta y_2/\Delta u_1\right]_{y_1}} \tag{3.10}$$

$$\lambda_{22}=\frac{\left(K_{22}\right)_{u_1}}{\left(K_{22}\right)_{y_1}} \tag{3.11}$$

$$\lambda_{22}=\frac{\left[\Delta y_2/\Delta u_2\right]_{u_1}}{\left[\Delta y_2/\Delta u_2\right]_{y_1}} \tag{3.12}$$

From above expressions it seems Determination of relative gain array is very difficult task but in general it not much difficult task because element of relative gain array has following property.

(1) The sum of element in each column is unity

(2) The sum of element in each row is unity

Thus for two input two output system if we calculate one element of relative gain array then all other three can calculated easily by relation.

$$\lambda_{12} = 1 - \lambda_{11} \tag{3.13}$$

$$\lambda_{21} = \lambda_{12} \tag{3.14}$$

And $$\lambda_{22} = \lambda_{11} \tag{3.15}$$

The above procedure to determine the element of RGA is experimentally. however if we have steady state model of system then it can calculated analytically.

For

$$y_2 = (K_{21}u_1) + (K_{22}u_2) \tag{3.16}$$

And $$y_2 = (K_{21}u_1) + (K_{22}u_2) \tag{3.17}$$

Where,

K_{11}, K_{12}, K_{22} and K_{21} are steady state gain of system

Eliminating u_2 from above equation we have

$$y_1 = (K_{11}u_1) + \frac{K_{21}(y_2 - K_{21}u_1)}{K_{22}} \tag{3.18}$$

Differentiating above equation with respect to u_1 we get

$$(K_{11})_{y_2} = \left(\frac{\Delta y_1}{\Delta u_1}\right)_{y_2} \tag{3.19}$$

$$= \frac{(K_{11} - K_{21}K_{12})}{K_{22}} \tag{3.20}$$

Thus the relative gain λ_{11} is given by

$$\lambda_{11} = (K_{11})_{u_2} \Big/ (K_{11})_{y_2} \tag{3.21}$$

$$= \frac{1}{1 - \frac{(K_{12}K_{21})}{(K_{11}K_{22})}} \tag{3.22}$$

3.3 Selection of control loop using RGA matrix

If RGA is constructed then the following case may be arise

3.3.1 If λ_{11}=0 , this means the value of diagonal element is zero and non diagonal element value is unity. And system can control by pairing y_1 with u_2 and y_2 with u_1 and resulting system in non- interaction with each other.

3.3.2 If λ_{11}= 1, this means system is non interaction with each other and pairing can be done like $y_1 - u_1$ and $y_2 - u_2$ and neither u_1 can be used to control y_2 nor u_2 can be used to control y_1.

3.3.3 If λ_{11}=0.5, this is the worst case and in this case both manipulated variable u_1 and u_2 affect the by same factor and degree of interaction in this case will be same .

3.3.4 If λ_{11} in the range between 0 to 0.5 (let 0.25) it means the value of diagonal element is 0.25 while the value of non diagonal element have 0.75 so more suitable pairing is y_1 with u_2 and y_2 with u_1.

3.3.5 If λ_{11} in the range between 0.5 to 1 (let 0.75) it means the value of diagonal element is 0.75 while the value of non diagonal element have 0.25 so most suitable pairing is $y_1 - u_1$ and $y_2 - u_2$.

3.3.6 If λ_{11} is greater than 1 it means non diagonal element have negative value, this means change in y_1 due change in first manipulated variable is reduce if loop between second output and second manipulated variable is closed and controlled response is affected by this interaction larger the of this means larger the effect of interaction .

3.4 De-coupler design for two tank interacting system

Dynamic behaviour of two tank interacting system can be studied by step change in flow rate and by observing the response of the system by developing a mathematical model of system.

Mathematical model of two tank interacting system-

Two tank interacting system is shown below the mathematical model is derived by assuming that

(1) Flow resistance is linear
(2) Both tank have equal and uniform cross section area
(3) And fluids are incompressible i.e density of fluid is constant.

Level of Tank 1 is depends on tank 2 level and level of tank 2 is depend upon level of tank 1 as a result of the interconnecting stream with flow rate $\boldsymbol{q_1}$. The term intaracting is referred as loading. The second tank of Fig.3.1 is said to *load* the first tank.

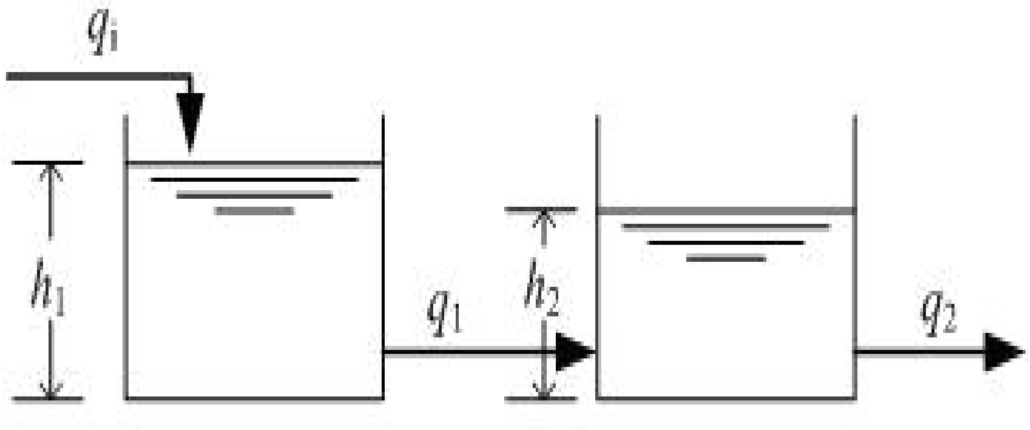

Fig3.1. Interacting two tank system

Applying mass balance around tank 1 we get

$$A_1 \frac{dh_1}{dt} = Q_i - Q_1 \tag{3.23}$$

Applying Mass balance around tank 2:

$$A_2 \frac{dh_2}{dt} = Q_1 - Q_2 \tag{3.24}$$

where:

A_1, A_2 - is the cross section area of the tank

Q_i - is the input flow to the first tank

Q_1 – is the output flow of the first tank and input flow to the second tank

Q_2 – is the output flow of first tank

$$Q_1 = \frac{H_1 - H_2}{R_1} \quad (3.25)$$

and $$Q_2 = \frac{H_2}{R_2} \quad (3.26)$$

Then the equation 3.23 and 3.24 reduce to

$$A_1 R_1 \frac{dh_1}{dt} + h_1 - h_2 = R_1 Q_i \quad (3.27)$$

$$A_2 R_2 \frac{dh_2}{dt} + \left(1 + \frac{R_2}{R_1}\right) h_2 - \frac{R_2}{R_1} h_1 = 0 \quad (3.28)$$

$$h_1 - h_2 = R_1 Q_i \quad (3.29)$$

$$\left(1 + \frac{R_2}{R_1}\right) h_2 - \frac{R_2}{R_1} h_1 = 0 \quad (3.30)$$

From the equation (3.26) and (3.27) steady state equivalents are

$$A_1 R_1 \frac{dh'_1}{dt} + h'_1 - h'_2 = R_1 Q'_i \quad (3.31)$$

$$A_2 R_2 \frac{dh'_2}{dt} + \left(1 + \frac{R_2}{R_1}\right) h'_2 - \frac{R_2}{R_1} h'_1 = 0 \quad (3.32)$$

Where

$$Q_i' = (Q_i - Q_{i,s}) \quad (3.33)$$

$$h_2' = (h_2 - h_{2,s}) \tag{3.34}$$

Taking the laplace transform of equation (3.30) and (3.31) we get:

$$H'_1(s) = \frac{(A_2R_2R_1)s + (R_2 + R_1)}{S^2 A_1R_1A_2R_2(A_1R_1 + A_2R_2 + A_1R_2)S + 1} Q_{i(s)} \tag{3.35}$$

$$\frac{H'_2(s)}{Q_{i(s)}} = \frac{(R_2)}{S^2 A_1R_1A_2R_2(A_1R_1 + A_2R_2 + A_1R_2)S + 1} \tag{3.36}$$

$$\frac{H'_2(s)}{Q'_i(s)} = \frac{(R_2)}{S^2 \tau_1\tau_2(\tau_1 + \tau_2 + A_1R_2)S + 1} \tag{3.37}$$

where:

$\tau_1 = A_1R_1$ and $\tau_2 = A_2R_2$

Consider a two tank interacting system as shown in figure below in which we have two manipulated variable r_1 and r_2 and now we have to calculate gain of each manipulated variable for each controlled variable Y_1 and Y_2 therefore we have to calculate four gain.

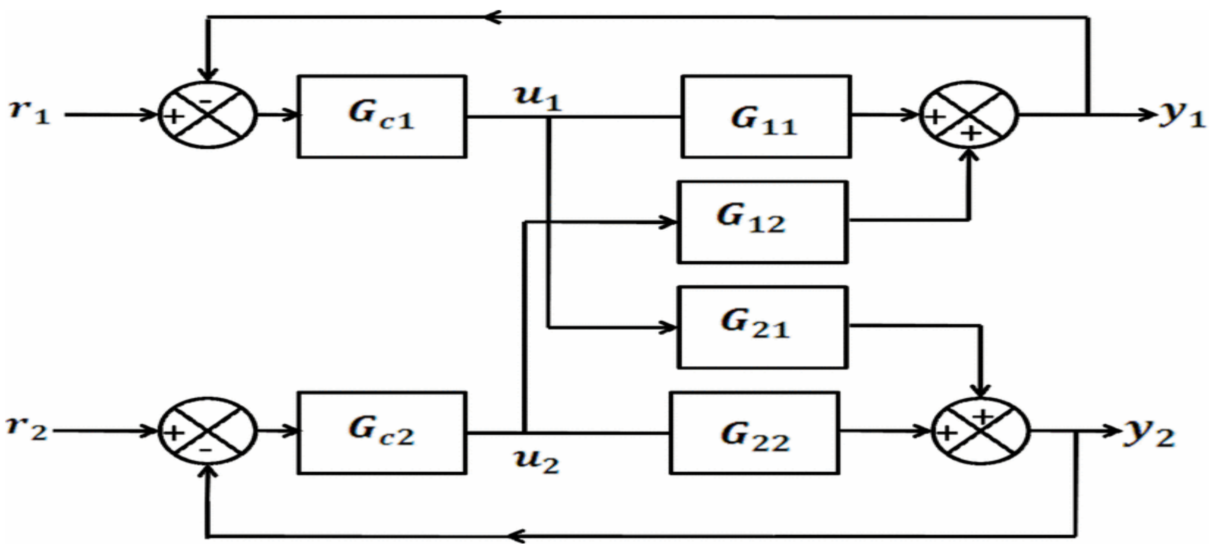

Fig. 3.2 Interacting two tank system

The gain of each manipulated variable for each controlled variable

$$\lambda_{11} = \left(\frac{\Delta h_1}{\Delta q_i}\right)_{q_2} \quad (3.38)$$

$$\lambda_{12} = \left(\frac{\Delta h_1}{\Delta q_2}\right)_{q_i} \quad (3.39)$$

$$\lambda_{21} = \left(\frac{\Delta h_2}{\Delta q_i}\right)_{q_2} \quad (3.40)$$

$$\lambda_{22} = \left(\frac{\Delta h_2}{\Delta q_2}\right)_{q_i} \quad (3.41)$$

De-coupler design for two tank interacting system

Block diagram decoupled two tank system is shown in figure below in which G_{c1} and G_{c2} are controller. d_{11}, d_{12}, d_{21} and d_{22} are coefficient of de-coupler

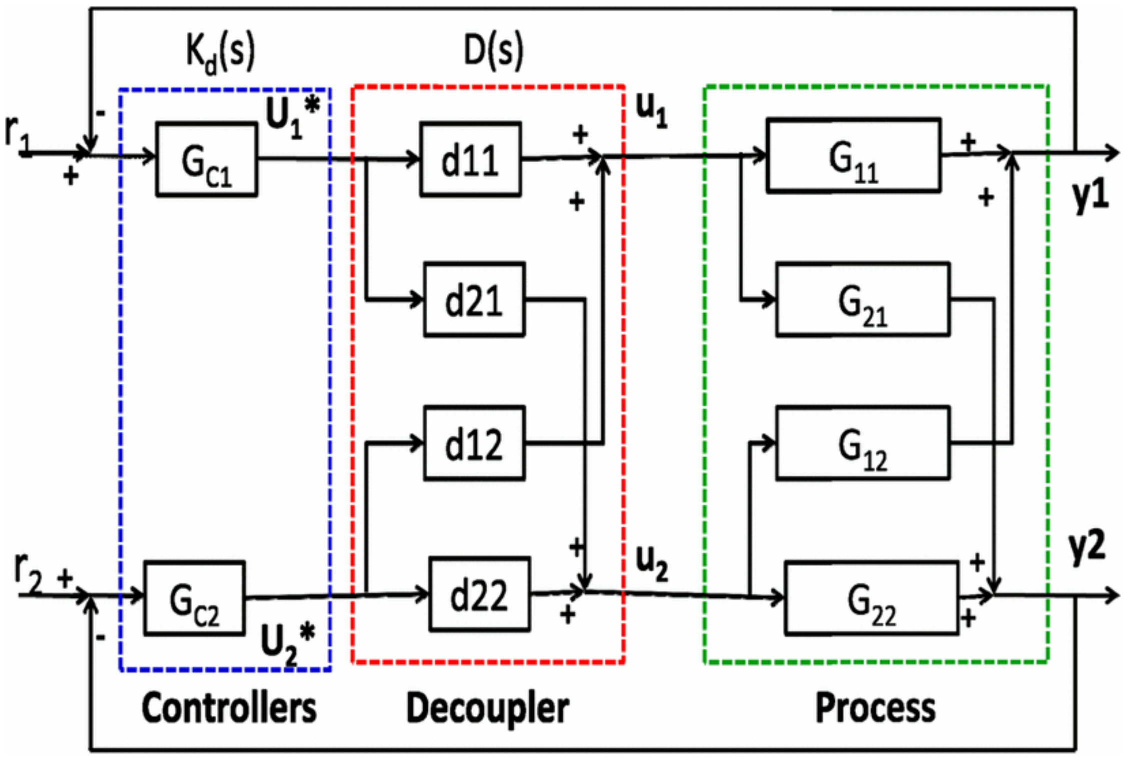

Fig.3.3 decoupled two tank interacting system

One of the important characteristics of the De-coupler circuit is that it change the manipulated variable irrespective of the other loop. The de-coupler work as feedback controller for first tank and work as feed forward controller for the second tank. To reduce the interaction between the two tank we have to implement two de-coupler circuit having Transfer function$D_{12}(s)$ and $D_{21}(s)$. Transfer function of decoupled system is given below.

$$D_{12}(s) = -\frac{G_2(s) * G_{12}(s)}{G_1(s) * G_{11}(s)} \tag{3.42}$$

$$D_{21}(s) = -\frac{G_1(s) * G_{21}(s)}{G_2(s) * G_{22}(s)} \tag{3.43}$$

Decentralized PID controller for two tank interacting non linear system

Most of industrial process basically a multi input multi output system for such type of system loop interaction may be arise and to solve such type of problem we have to design de-coupler.

Relative gain array

The biggest advantage of relative gain array matrix is it is requires the minimal knowledge of plant or physical system like steady state gain another important advantage is independency of result on physical units and variable of process. The relative gain for i_{th} manipulated variable and j_{th} controlled variable is defined as

$$\lambda_{ij} = \frac{open\ loop\ gain\ between\ Uj\ and\ Yi}{closed\ loop\ gain\ between\ Uj\ and\ Yi}$$

Decoupling design for non linear two tank interacting system

Decoupling design is to calculate the coefficient which cancel the effect of interaction and allowing us to independent control loop in decentralized control objective is not to eliminate interaction but take it to consideration . objective in decoupling design is to compensate the effect of interaction which is come in to picture due to cross coupling of the process variable.

Basic block diagram of two input two output system is shown in figure below

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} X \begin{bmatrix} Gc_1 & 0 \\ 0 & Gc_2 \end{bmatrix} X \begin{bmatrix} r_1 - y_1 \\ r_2 - y_2 \end{bmatrix} \quad (3.44)$$

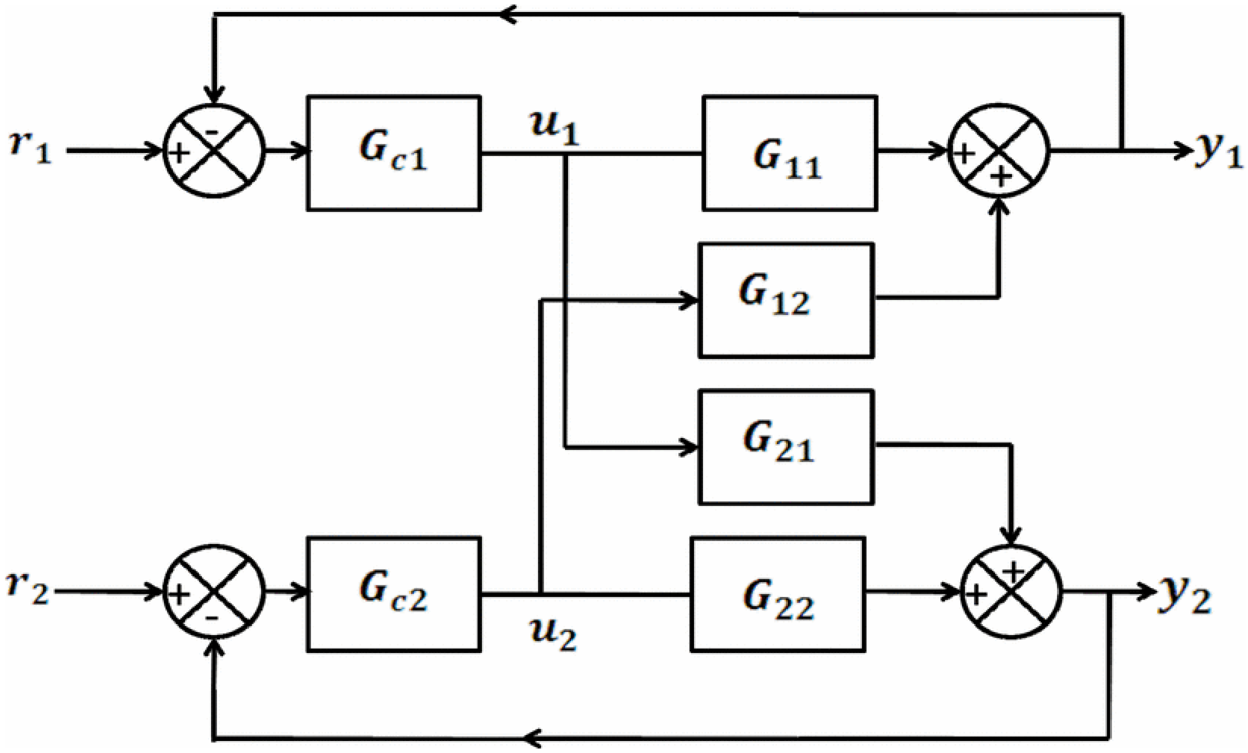

Fig.3.4 Block diagram of two input two output system

In this system interactions G_{12} and G_{21} is not zero we have to manipulate controller in this way that the effect of G_{12} and G_{21} is appears zero . this means output of controller is transformed with matrix which is contains decoupling function and thus manipulated variable is given by

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} X \begin{bmatrix} Gc_1 & 0 \\ 0 & Gc_2 \end{bmatrix} X \begin{bmatrix} r_1 - y_1 \\ r_2 - y_2 \end{bmatrix} \quad (3.45)$$

And the coefficient of the $d_{11,}d_{12,}d_{21}$ and d_{22} is given by

$$d_{11} = \left(\frac{G_{22} X H_1}{G_{11} G_{22} - G_{21} G_{12}} \right) \tag{3.46}$$

$$d_{12} = -\left(\frac{G_{12} X d_{22}}{G_{11}} \right) \tag{3.47}$$

$$d_{22} = \left(\frac{G_{11} X H_2}{G_{11} G_{22} - G_{21} G_{12}} \right) \tag{3.48}$$

And $$d_{21} = -\left(\frac{G_{21} X d_{11}}{G_{22}} \right) \tag{3.49}$$

Assume

$$H_1 = \left(\frac{(G_{22} X G_{11}) - (G_{21} X G_{12})}{G_{22}} \right) \tag{3.50}$$

$$H_2 = \left(\frac{(G_{22} X G_{11}) - (G_{21} X G_{12})}{G_{11}} \right) \tag{3.51}$$

The element of the two input two output system for decoupling matrix which is try to eliminate the effect of interaction from loop is given by

$$d_{12}(s) = -\left(\frac{(G_{12}(s))}{G_{11}(s)} \right) \tag{3.52}$$

$$d_{21}(s) = -\left(\frac{(G_{21}(s))}{G_{22}(s)} \right) \tag{3.53}$$

And $d_{11}(s) = d_{22}(s) = 1$

General block diagram of two input two output system with decoupling and controller for a single loop is shown below.

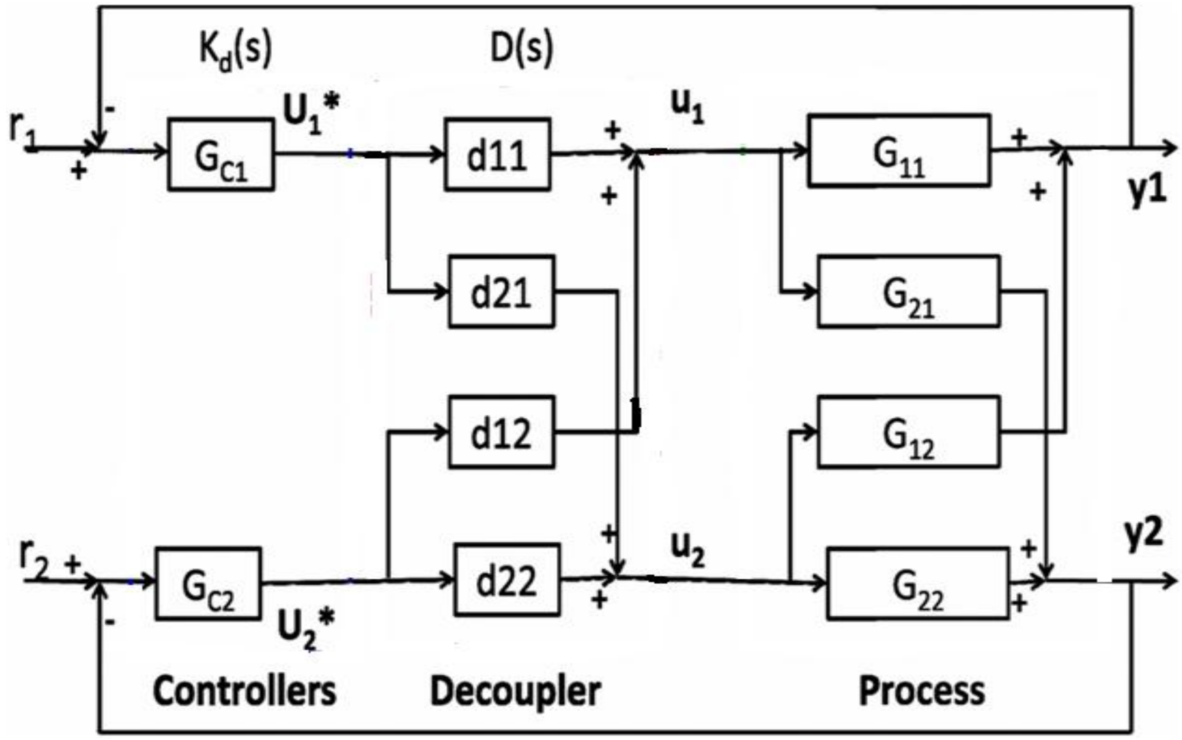

Fig. 3.5 General block diagram of two input two output system with decoupling and controller for a single loop.

Mathematical modelling of two tank conical interacting system

Two tank interacting conical system is shown in figure below it is consist of two identical conical tank each having equal area and equal height in this system two pump which independently deliver flow F_{in1} and F*in2* two the tank 1 and tank 2 respectively through the control valveCV_1 and CV_2 both of the tank connect through manual control valve MV_{12} at bottom as shown in figure below

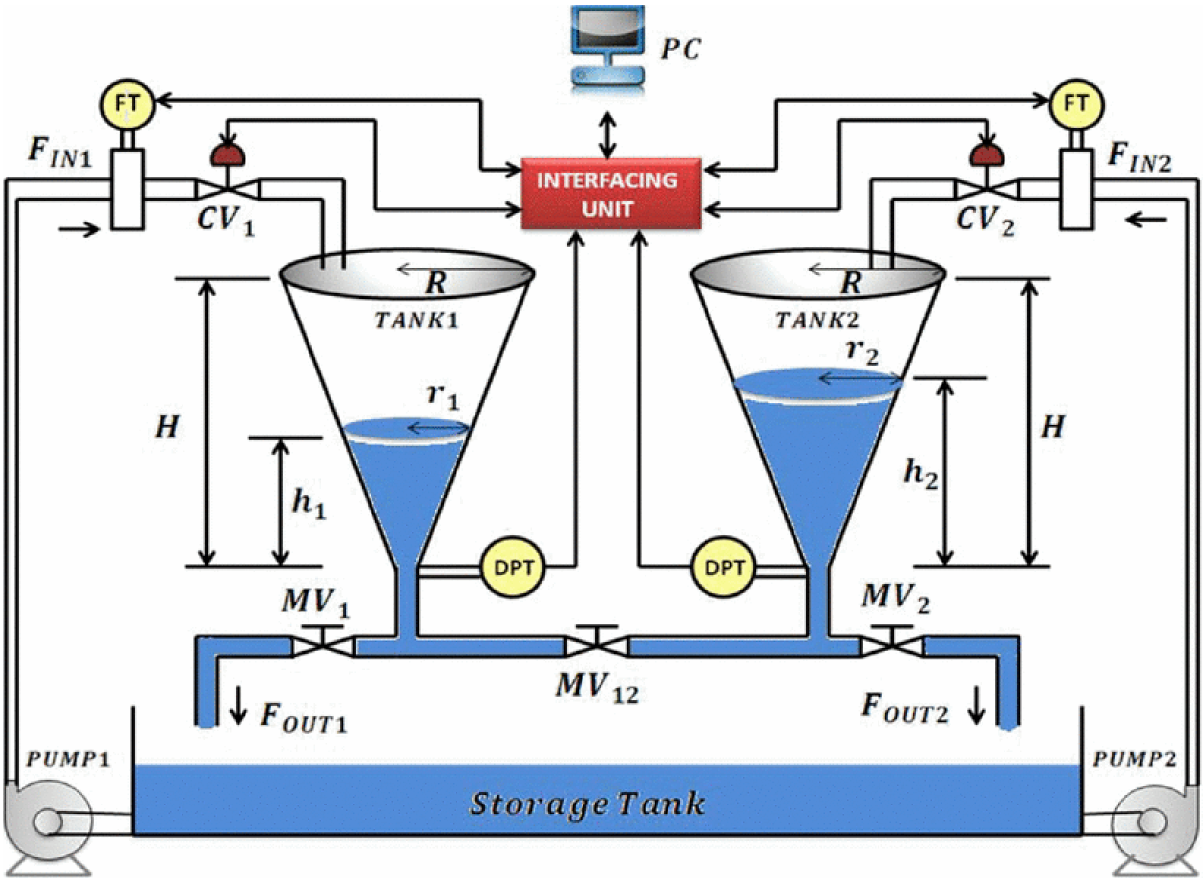

Fig 3.6 two tank conical interacting system

Table 3.1 Various operating parameter of two tank conical interacting system.

Parameter	Description	Value
R	Top radius of conical tank	19.25 cm
H	Maximum height of TANK1 & TANK2	73 cm
F_{IN1} & F_{IN2}	Maximum Inflow to TANK1 & TANK2 respectively	111.11 cm^3/s
β_1	Valve co-efficient of MV_1	35 cm^2 / s
β_{12}	Valve co-efficient of MV_{12}	78.28 cm^2 / s
β_2	Valve co-efficient of MV_2	19.69 cm^2 / s

In this system two tank conical interacting system is considered as two input two output process in which level in first and second tank is considered as controlled variable while flow in first and second tank is considered as manipulate variable. The mathematical model of two tank conical interacting system is given by

$$\frac{dh_1}{dt} = \left(\frac{Fi_1 - h_1 \frac{dA(h_1) - \left(\beta_1 X \sqrt{h_1}\right) - sign(h_1 - h_2) X \beta_{12} \sqrt{(h_1 - h_2)}}{dt}}{\frac{\pi r^2 {h_1}^2}{3H^2}} \right) \tag{3.54}$$

$$\frac{dh_2}{dt} = \left(\frac{Fi_2 + sign(h_1 - h_2) \beta_{12} \sqrt{(h_1 - h_2)} - \beta_2 \sqrt{h_2} - h_2 \frac{dA(h_2)}{dt}}{\frac{\pi r^2 {h_2}^2}{3H^2}} \right) \tag{3.55}$$

Where

Gc_2 is the area of first tank at given height (h_1)(cm2)

$A(h_2) =$ is the area of first tank at given height (h_2)(cm2)

h_1 = is the height of liquid level in first tank (cm)

h_2 = is the height of liquid level in second tank (cm)

To model the two tank conical interacting system transfer function matrix model is taken and it can be given as

$$\begin{bmatrix} h_1(s) \\ h_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$

Where G_{ij} is considered as first order system with delay the generalized form can be given as

$$G_{ij}(\mathrm{s}) = \frac{k_{ij} e^{-sTd}}{S\tau_{ij} + 1} \tag{3.56}$$

The transfer function parameter for two tank interacting system is given below

	K	τ (Seconds)	T_d(Seconds)
$G_{11}(s)$	0.1365	487.5	117.5
$G_{12}(s)$	0.11075	705	154
$G_{21}(s)$	0.0905	822	92
$G_{22}(s)$	0.129	715.5	158.5

The relative gain array matrix for two tank conical interacting system is given below

$$\Lambda = \begin{array}{c c} & \begin{array}{cc} h_1 & h_2 \end{array} \\ \begin{array}{c} F_{IN1} \\ F_{IN2} \end{array} & \begin{bmatrix} 2.32 & -1.32 \\ -1.32 & 2.32 \end{bmatrix} \end{array}$$

From the above calculation it is clear that the dominant variable that are to be paired as input flow to first tank as level of first tank and input flow to second tank as level in second tank

The coefficient of de-coupler for two tank interacting system is given below

$$d_{11}(s) = d_{22}(s) = 1$$

$$d_{12}(s) = \frac{(-395.6819641s - 0.811355311)e^{-36.5s}}{705s+1}$$

$$d_{21}(s) = \frac{(-501.9593019s - 0.701550387)e^{-66.5s}}{822s+1}$$

Tuning of two tank conical interacting non linear system is done using Ziegler Nichols tuning method and the coefficient of controller is given table below

S.No	Controller	K_p	τ_i	τ_d
1.	Gc_1	4.98	235	292.75
2.	Gc_2	5.4	317	79.25

CHAPTER: 4

PERFORMANCE ANALYSIS AND COMPARISION OF THREE TANK SYSTEM

4.1 Performance Analysis of three tank level control using feedback and feed forward-feedback controller

Control of liquid in process control industries is important in order to provide desired performance because level of system can affect the pressure and rate of flow in the industry So it is important to maintain level at desired set point. in this section we are considering three tank system which are connected in different configuration like interacting and non interacting

4.2 CASE (1) - Non-Interacting Three tank system

Non-Interacting three tank system is shown in figure below in which manipulated variable is input flow rate to the first tank and controlled variable is level in the third tank in this system we are also considering the effect of disturbance on the response of the system. To minimize the effect of these disturbances we have to implement Feed-forward controller with suitable tuning parameter

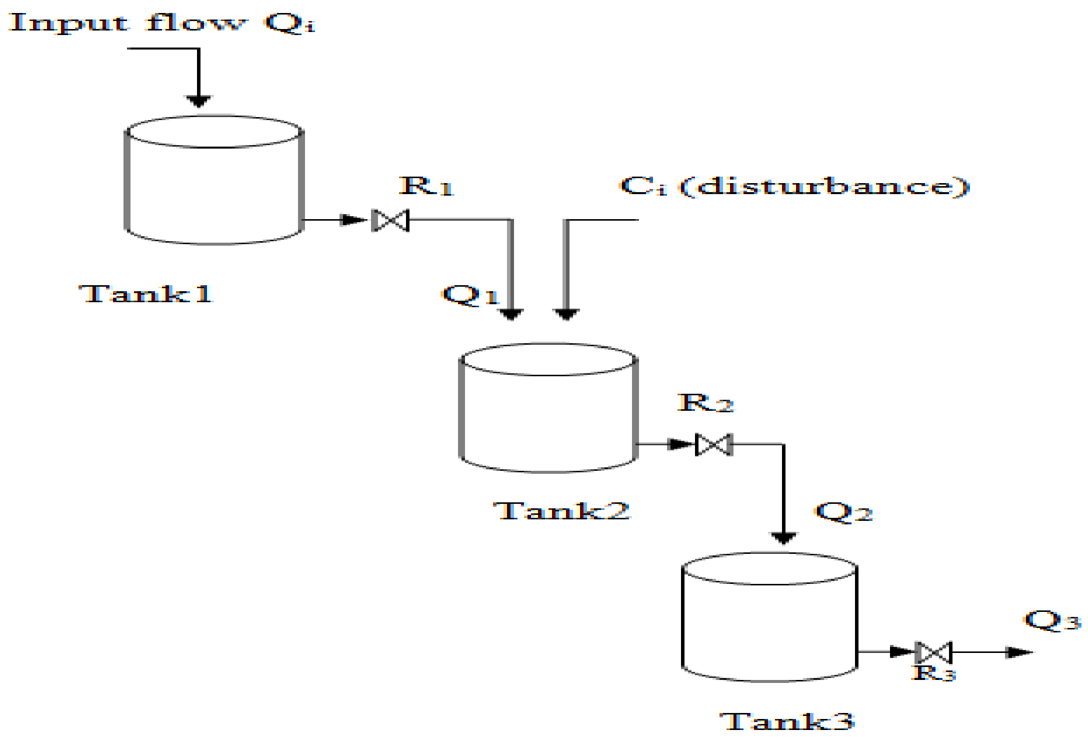

fig.4.1 Three tank Non-Interacting system

in this system output flow of first tank is input flow of second tank and similar way second tank is connected to third tank.

4.2.1 Mathematical modelling of Three tank Non-Interacting system

Applying Mass Balance around first tank we have

$$A_1 \frac{dh_1}{dt} = (Q_i - Q_1) \tag{4.1}$$

From Valve relation we have

$$Q_1 = \left(\frac{H_1}{R_1}\right) \tag{4.2}$$

Where,

Q_i - is the inlet flow rate to first tank (m^3/s)

Q_1 - is the outlet flow rate of first tank (m^3/s)

R_1 - is Outlet flow rate resistance of first tank ($m/m^3/s$)

A_1 - is the area of first tank (m^2)

H_1 - is the Actual liquid level in first tank (m)

Mass Balance around tank 2

$$A_2 \frac{dh_2}{dt} = (Q_1 - Q_2) \tag{4.3}$$

From the Valve relationship we have

$$Q_2 = \left(\frac{H_2}{R_2}\right) \tag{4.4}$$

Where**,**

Q_1 - is the outlet flow rate of first tank and inlet flow rate to second tank (m^3/s)

Q_2 - is the outlet flow rate of second tank (m^3/s)

R_2 - is the flow resistance of second tank ($m/m^3/s$)

A_2 - is the Area of second tank (m^2)

H_2 - is the Actual liquid level in the second tank (m)

Applying Mass Balance around the third tank

$$A_3 \frac{dh_3}{dt} = (Q_2 - Q_3) \tag{4.5}$$

From the Valve relationship we have

$$Q_3 = \left(\frac{H_3}{R_3} \right) \tag{4.6}$$

Where,

Q_2 - is the third Tank input flow (m^3/s)

Q_3 - is the third Tank output flow (m^3/s)

R_3 -is the flow resistance of third tank ($m/m^3/s$)

A_3 -is the Area of third tank (m^2)

H_3 - is the liquid level in the third tank (m)

Transfer function of three tank system is calculated by rearranging all above equation (4.1) to equation (4.6),

$$\frac{H_3(s)}{Q_i(s)} = \frac{R_3}{(A_3 R_3 s + 1)(A_2 R_2 s + 1)(A_1 R_1 s + 1)} \tag{4.7}$$

By Assuming,

$$R_1 = R_2 = R_3 = A_1 = A_2 = A_3 = 1.$$

Equation (4.7) is reduced to

$$\frac{H_3(s)}{Q_i(s)} = \left[\frac{1}{(s+1)^3}\right] \tag{4.8}$$

4.3 Interacting Three tank system case (1)

Interacting three tank system is shown in figure below in which tank 1and tank 2 are in interaction with each other while tank 2 and tank 3 are non interaction with each other

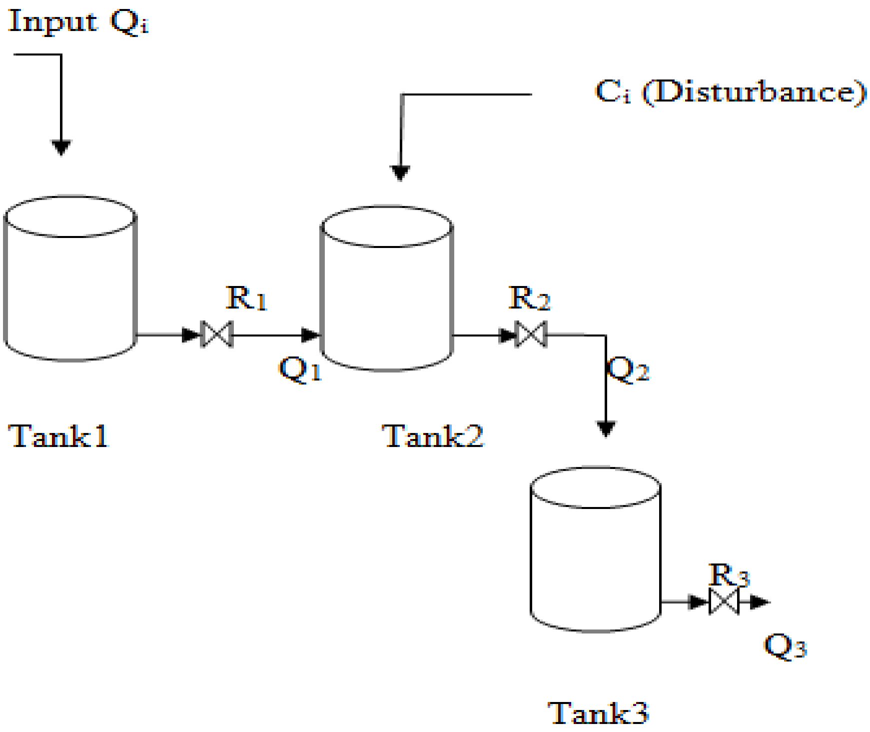

Fig.4.2 Interacting three tank system (case1)

By applying mass balance and valve relation to all three tank and rearranging them we get transfer function

$$\frac{H_3(s)}{Q_i(s)} = \frac{R_1R_2R_3}{((S\tau_1+1)(S\tau_2R_1+R_1+R_2)-R_2)(S\tau_3+1)} \tag{4.9}$$

By assuming,

$$R_1 = R_2 = R_3 = A_1 = A_2 = A_3 = 1$$

Above equation reduce to

$$\frac{H_3(s)}{Q_i(s)} = \frac{1}{((S+1)(S+2)-1)(S+1)} \tag{4.10}$$

4.4 Interacting three tank system case (2)

Interacting three tank system is shown in figure below in which tank 1 and tank 2 are non interaction with each other while tank 2 and tank 3 are interaction with each other

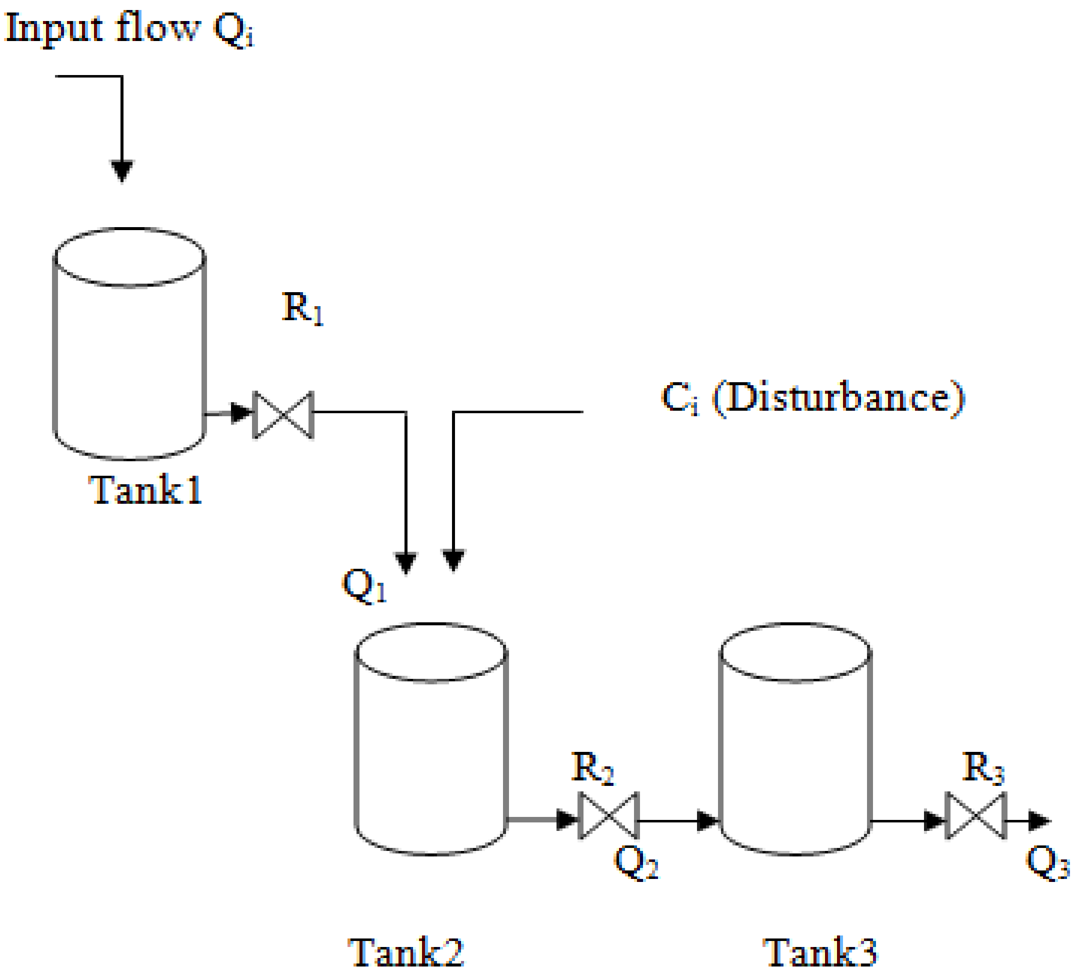

Fig.4.3 Interacting three tank system (case2)

By applying mass balance and valve relation to all three tank and rearranging them we get transfer function

$$\frac{\mathrm{C(s)}}{\mathrm{R(s)}}=\frac{R_1R_2R_3}{((\mathrm{S}\tau_2+1)(\mathrm{S}\tau_3R_2+R_2+\mathrm{R}_3)-\mathrm{R}_3)(\mathrm{S}\tau_1+1)} \tag{4.11}$$

By assuming

$$R_1=R_2=R_3=A_1=A_2=A_3=1$$

Above equation reduce to

$$\frac{H_3(\mathrm{s})}{Q_i(\mathrm{s})}=\frac{1}{((\mathrm{S}+1)(\mathrm{S}+2)-1)(\mathrm{S}+1)} \tag{4.12}$$

4.5 Disturbance analysis of three tank non-interacting system

In this case we are considering the disturbance is applied to second tank and transfer function between level of third tank and input disturbance to second tank is calculated

$$\frac{H_3(\mathrm{s})}{C_i(\mathrm{s})}=\frac{R_3}{(\mathrm{A}_3R_3\mathrm{s}+1)(\mathrm{A}_2R_2\mathrm{s}+1)} \tag{4.13}$$

4.6 Disturbance analysis of three tank interacting system (case1)

$$\frac{H_3(\mathrm{s})}{C_i(\mathrm{s})}=\frac{R_3}{(\mathrm{A}_3R_3\mathrm{s}+1)(\mathrm{A}_2R_2\mathrm{s}+1)} \tag{4.14}$$

4.7 Disturbance analysis of three tank interacting system (case2)

$$\frac{H_3(\mathrm{s})}{C_i(\mathrm{s})}=\frac{R_2}{A_1R_1A_2R_2\mathrm{s}^2+(A_1R_1+A_2R_2+A_1\mathrm{R}_2)\mathrm{s}+1} \tag{4.15}$$

CHAPTER: 5

Implementation and Result

5.1 MATLAB Implementation and Results

Response of two input two output process is shown in figure below in which relative gain array is calculated to obtain suitable pairing. After calculating relative gain the suitable pairing for two input two output process is level of first tank is controlled by first manipulated variable while level of second tank in controlled by second manipulated variable. after pairing we are designed the suitable de-coupler circuit to eliminate the effect of interaction in two input two output process. Finally two single loop controller is designed.

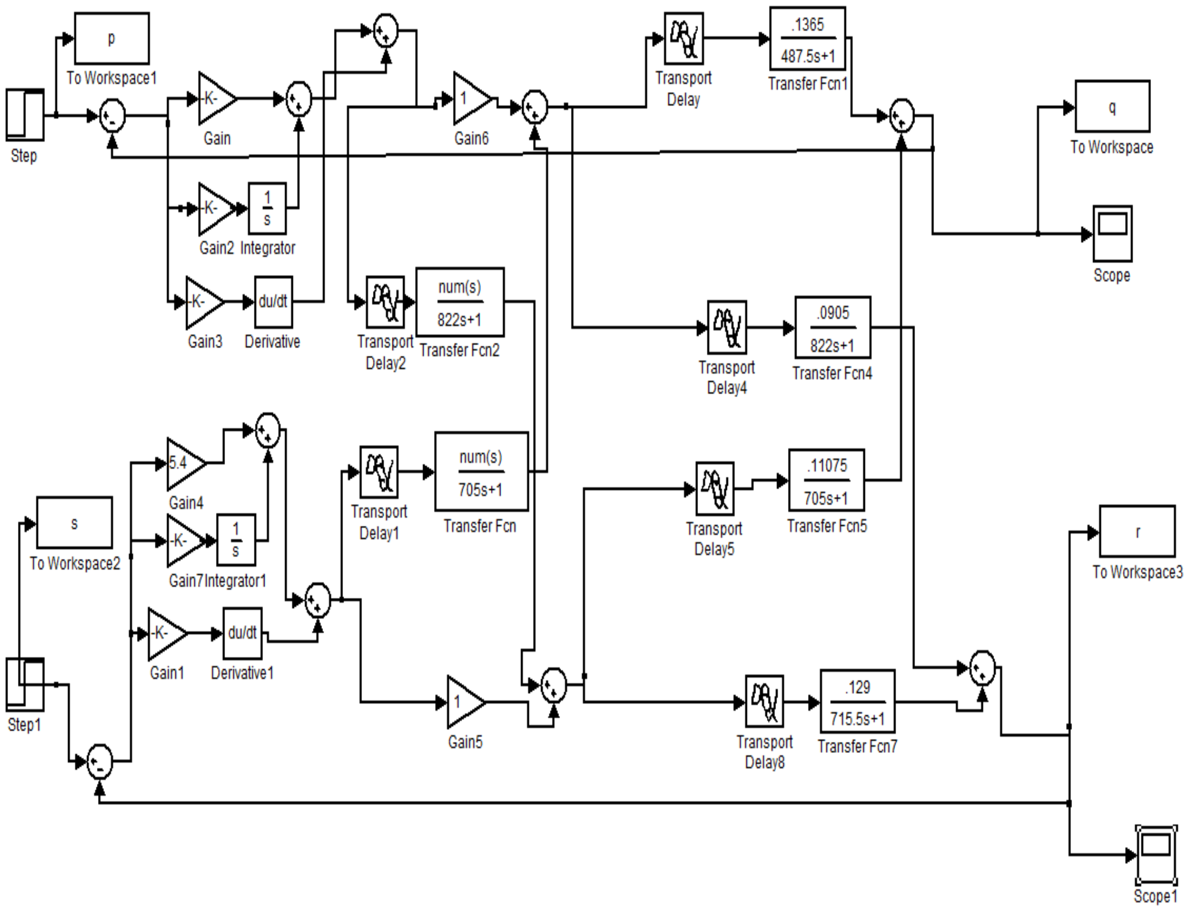

Fig.5.1 Decentralized PI Controller for Two input Two output process using Simulink.

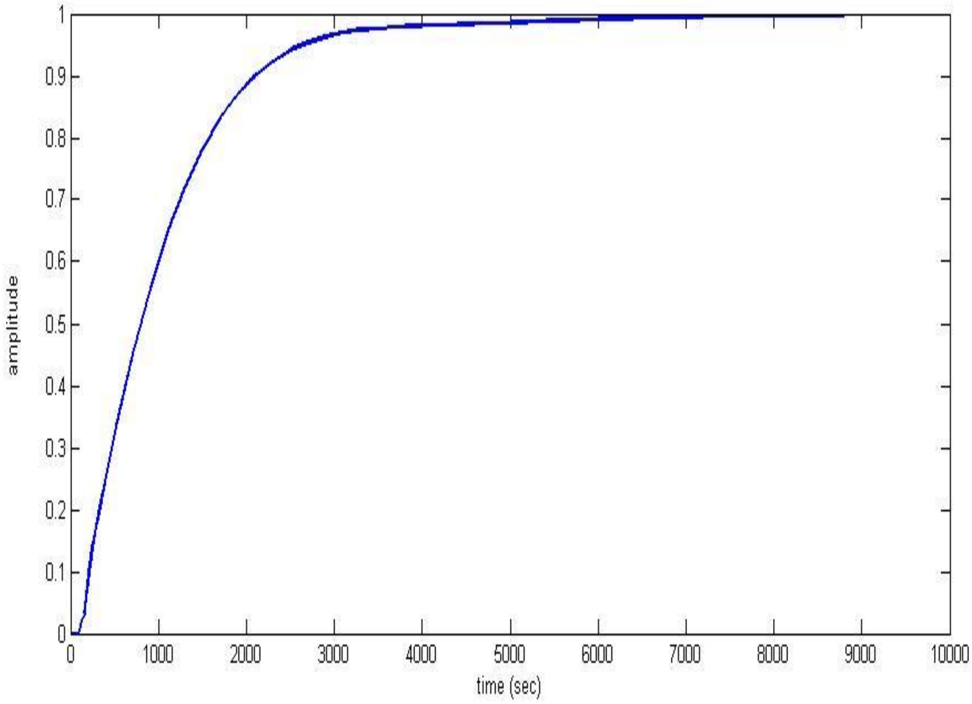

Fig. 5.2 Output response of the first tank using De-coupler with Step Input.

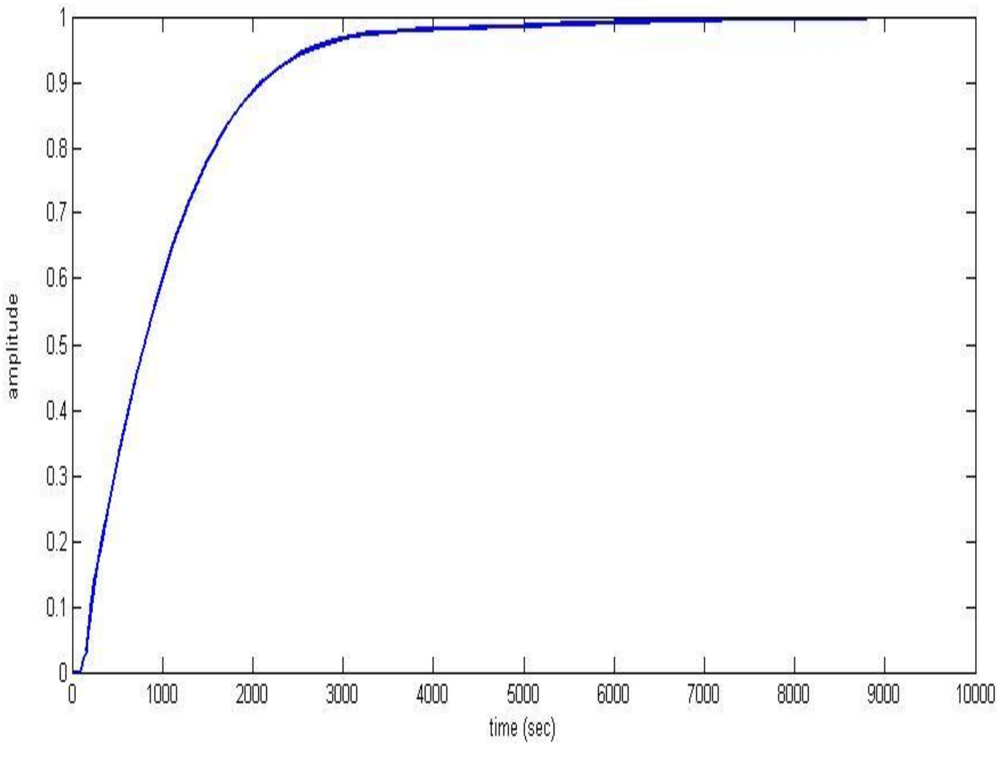

Fig. 5.3 Output response of the second tank using De-coupler with Step Input.

5.2 Feedback and Feed Forward- Feedback Controller for three tank system

5.2.1 Three tank Non-Interacting System

Response of Three Tank non-interacting system is shown in figure below in which transfer function between level of third tank and input flow rate is calculated and then corresponding PI controller is designed then disturbance is applied to second tank and transfer function between level of third tank and input disturbance is calculated then corresponding feed-forward controller is designed and finally response is observed with step input in simulink.

Overall transfer of three tank non interacting system is given below

$$\frac{H_3(s)}{Q_i(s)} = \frac{R_3}{(A_3 R_3 s+1)(A_2 R_2 s+1)(A_1 R_1 s+1)} \tag{5.1}$$

Transfer function with respect to disturbance

$$\frac{H_3(s)}{C_i(s)} = \frac{R_3}{(A_3 R_3 s+1)(A_2 R_2 s+1)} \tag{5.2}$$

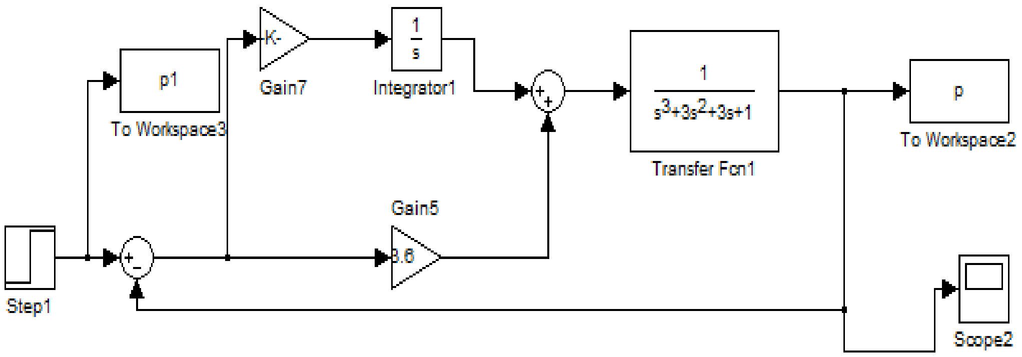

Fig.5.4 Feedback controller for three tank non interacting system.

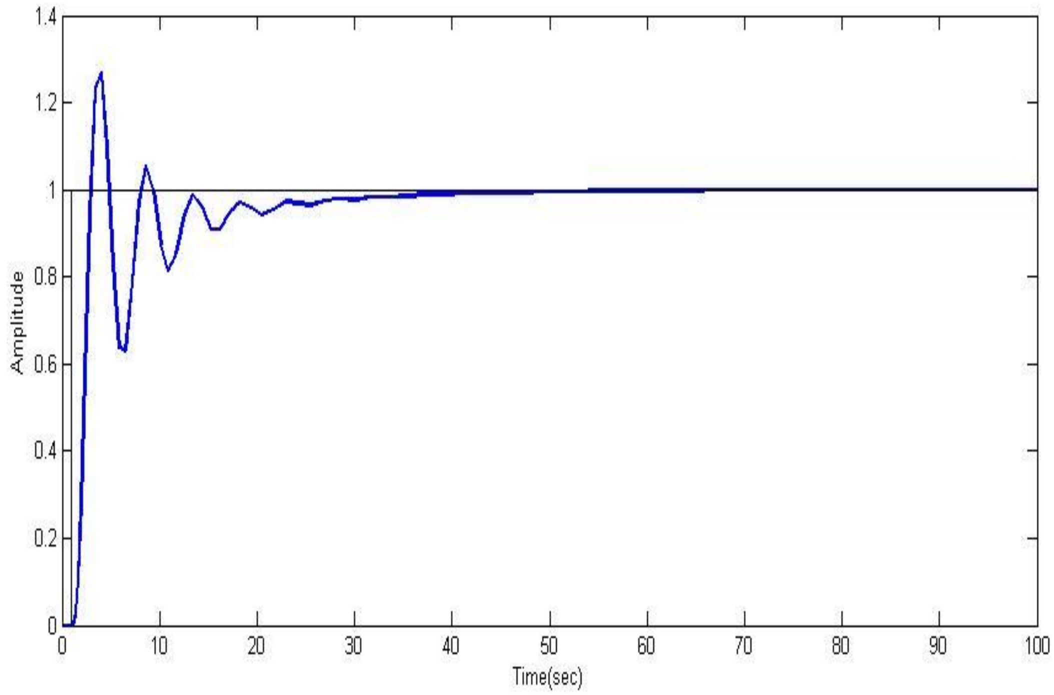

Fig. 5.5 Output response of feedback controller for three tank non interacting system.

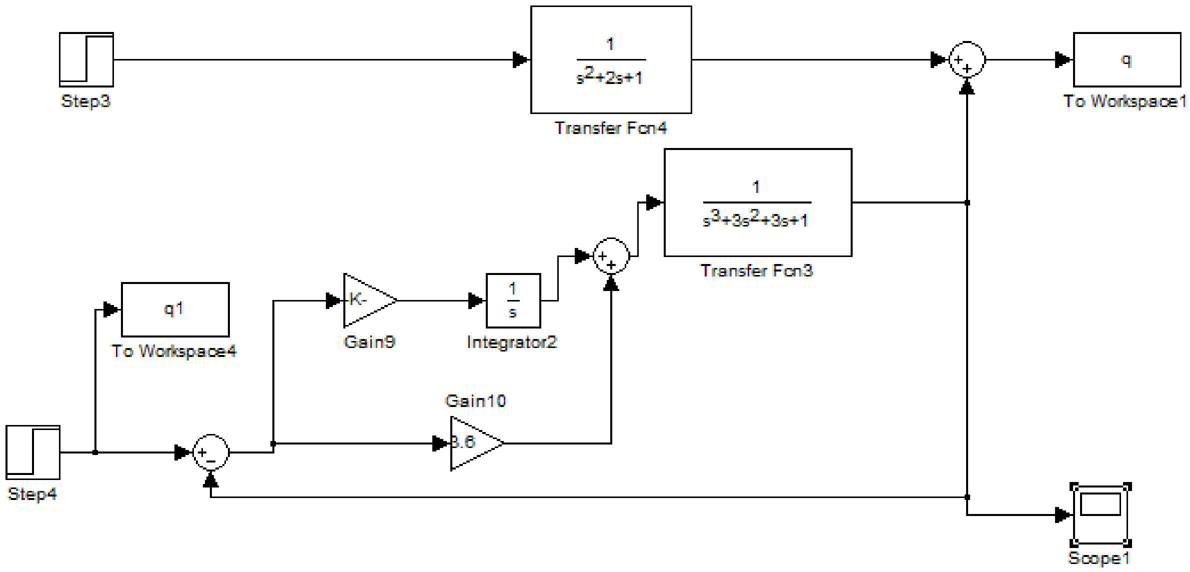

Fig. 5.6 Fee back controller for three tank non interacting system with disturbance.

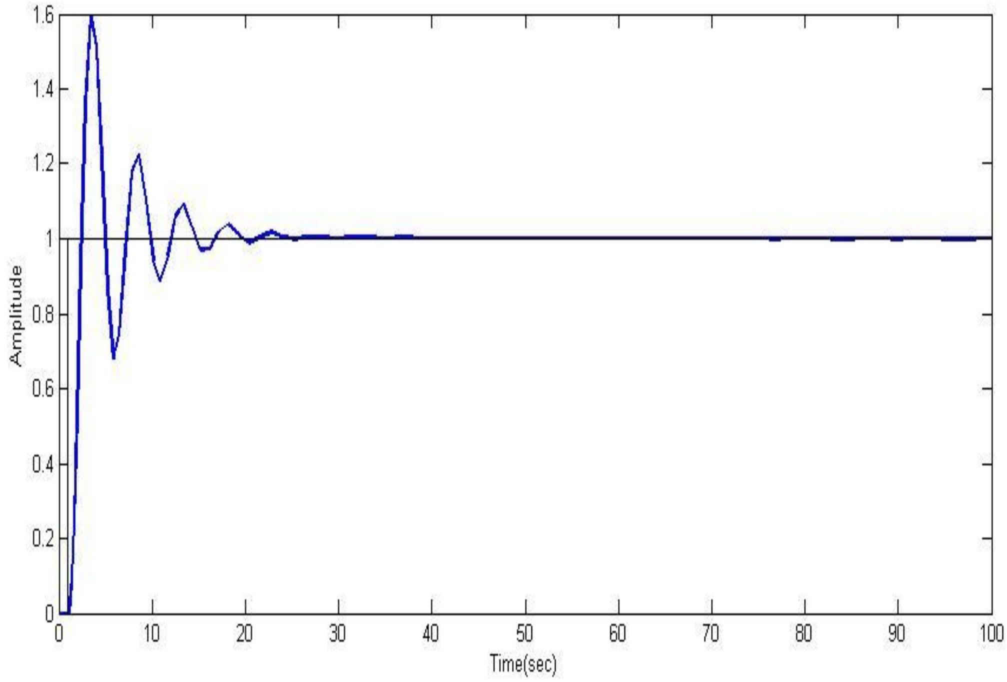

Fig.5.7 Output response of feedback controller for three tank non interacting system with disturbance.

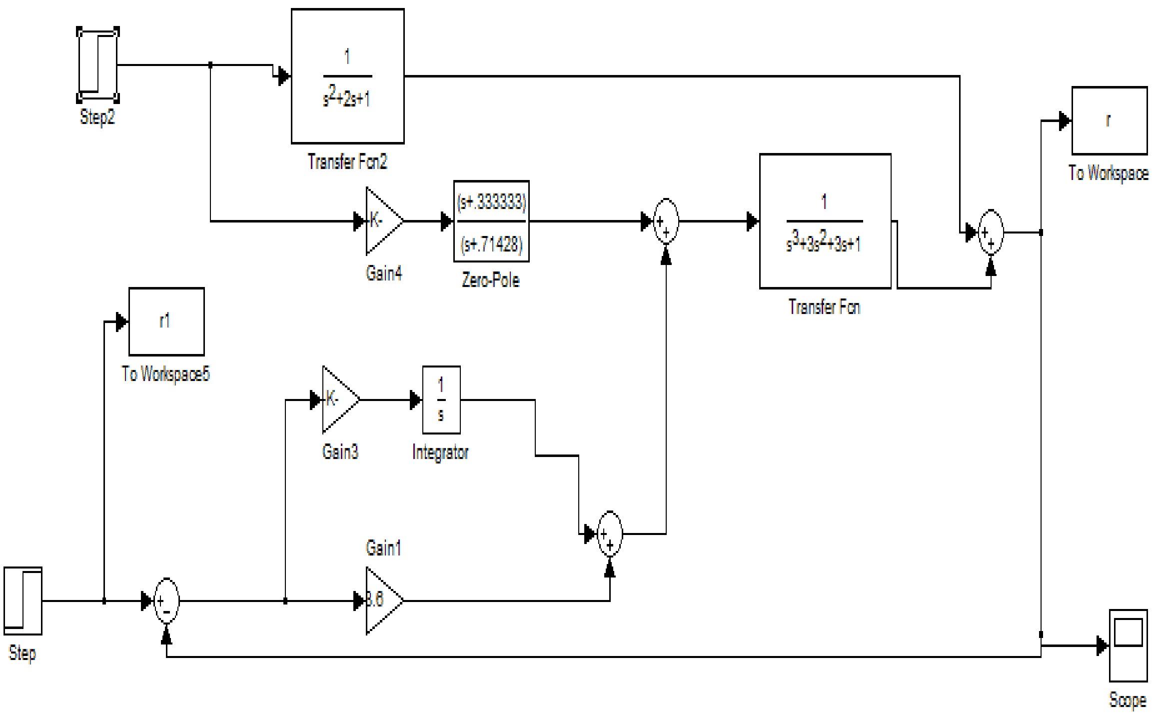

Fig. 5.8 Feedback and feedback-feed forward controller for three tank non interacting system with disturbance

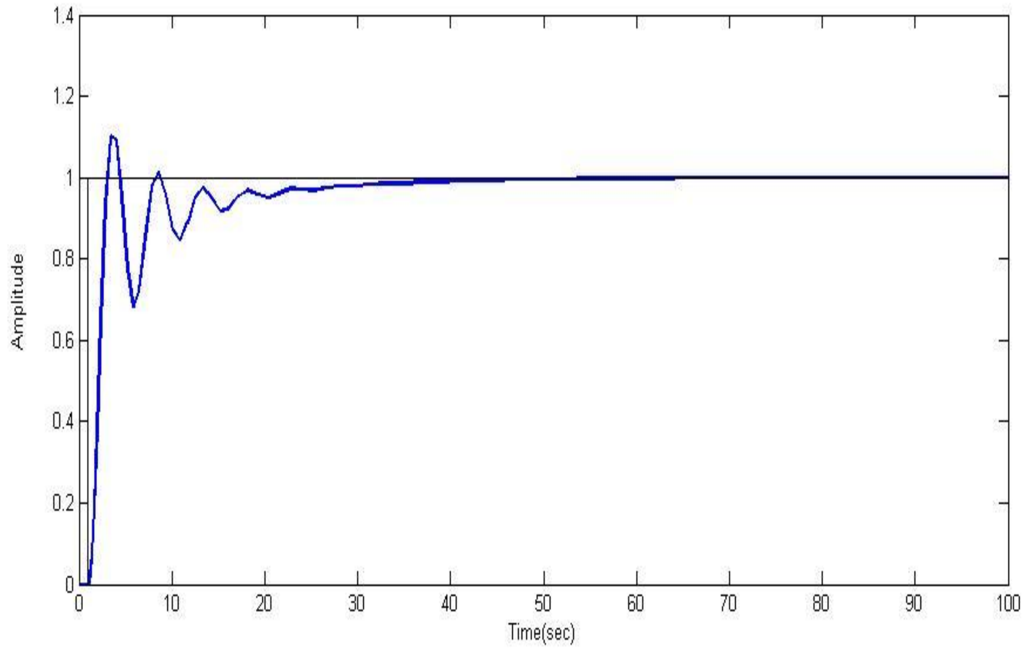

Fig.5.9 Output response of feedback and feed forward-feedback controller for three tank non interacting system with disturbance

5.2.2 Three tank interacting system (case1)

Response of Three tank interacting system is shown below in which first tank and second tank are connected in interacting with each other while second tank third tank are non interaction with each other the transfer function between level of third tank and input flow to the first tank is calculated and then corresponding PI controller is designed then disturbance is applied to the second tank and transfer function between level of third tank and input disturbance is calculated and Feed forward controller is designed.

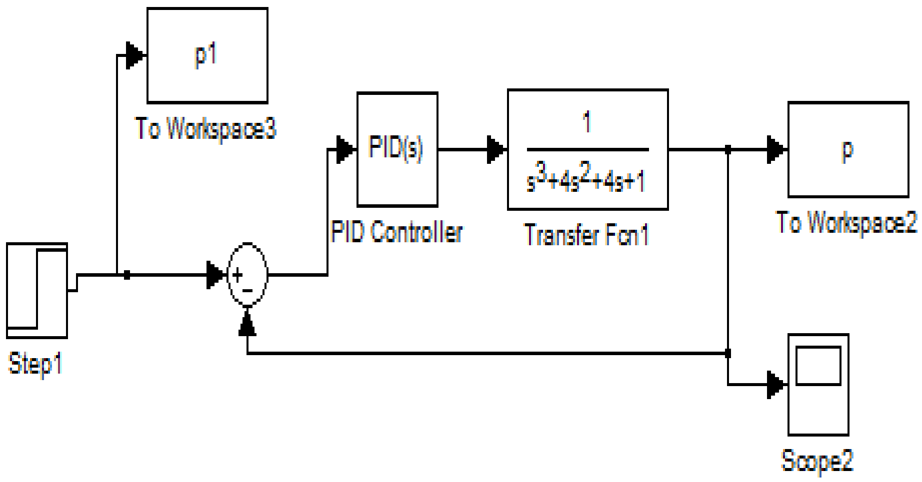

Fig.5.10 Feedback controller for three tank Interacting system(case1).

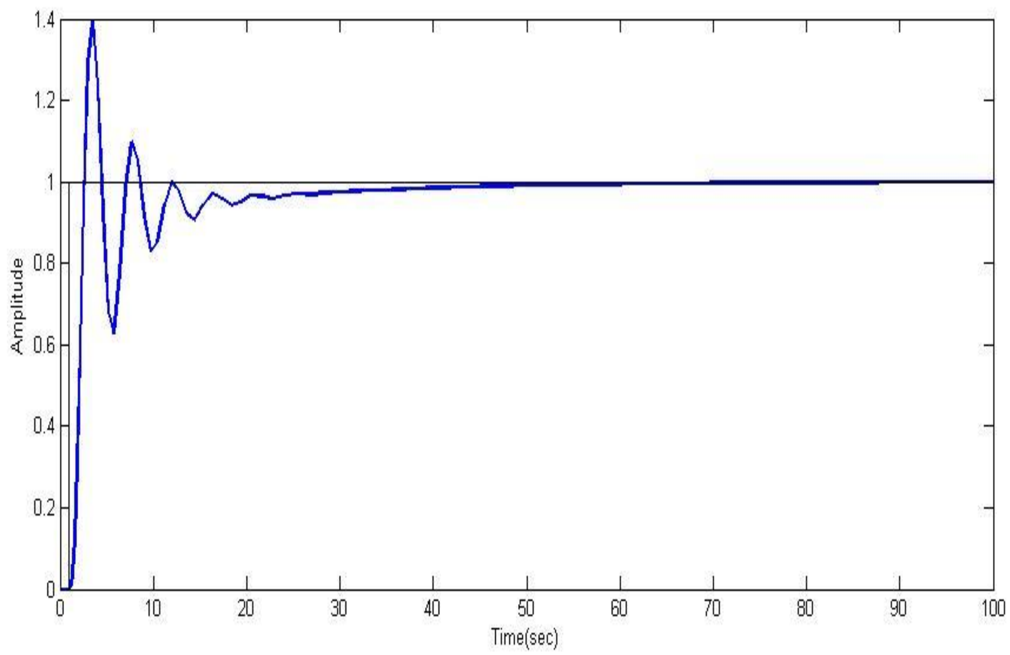

Fig.5.11 Output response of feedback controller for three tank Interacting system (case1).

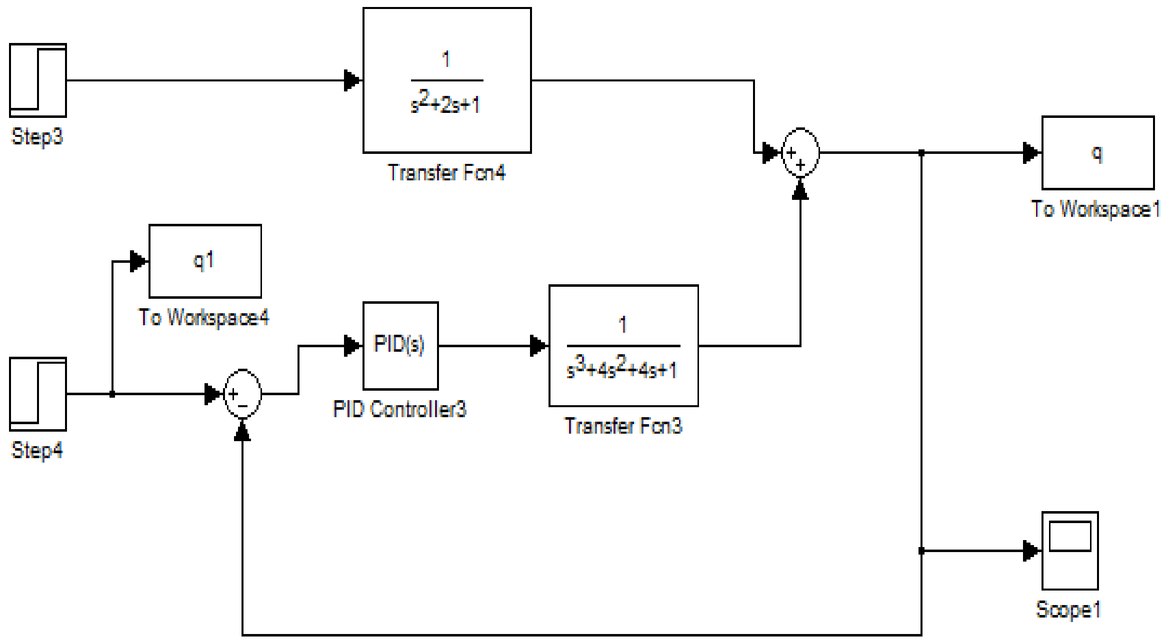

Fig.5.12 Feedback controller for three tank Interacting system (case1) with disturbance.

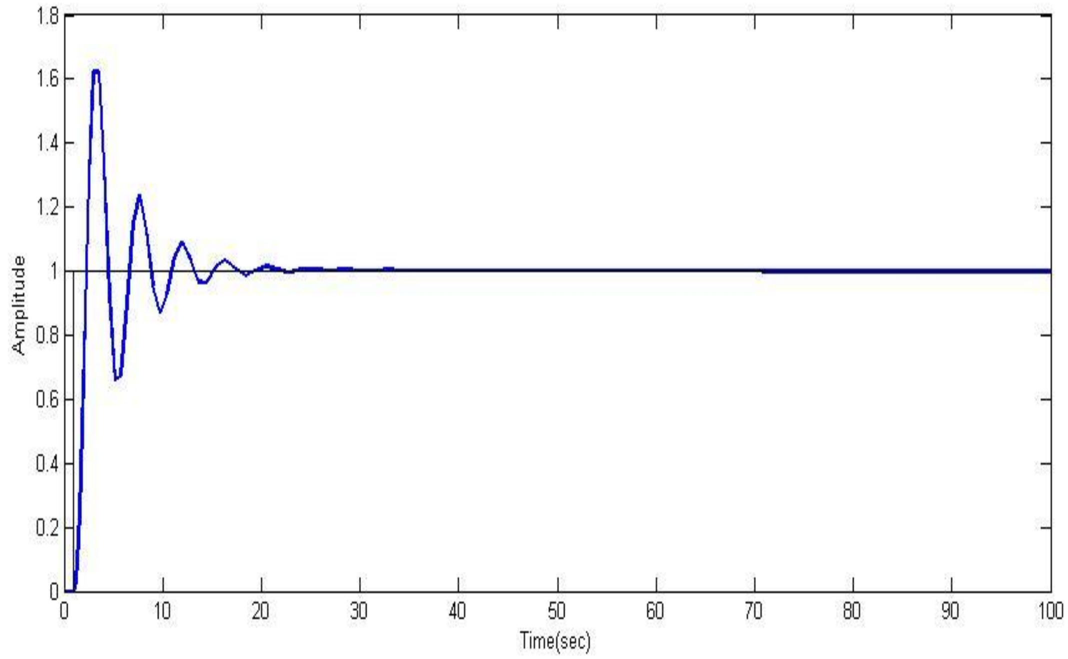

Fig.5.13 Output response of feedback controller for three tank Interacting system(case1) with disturbance.

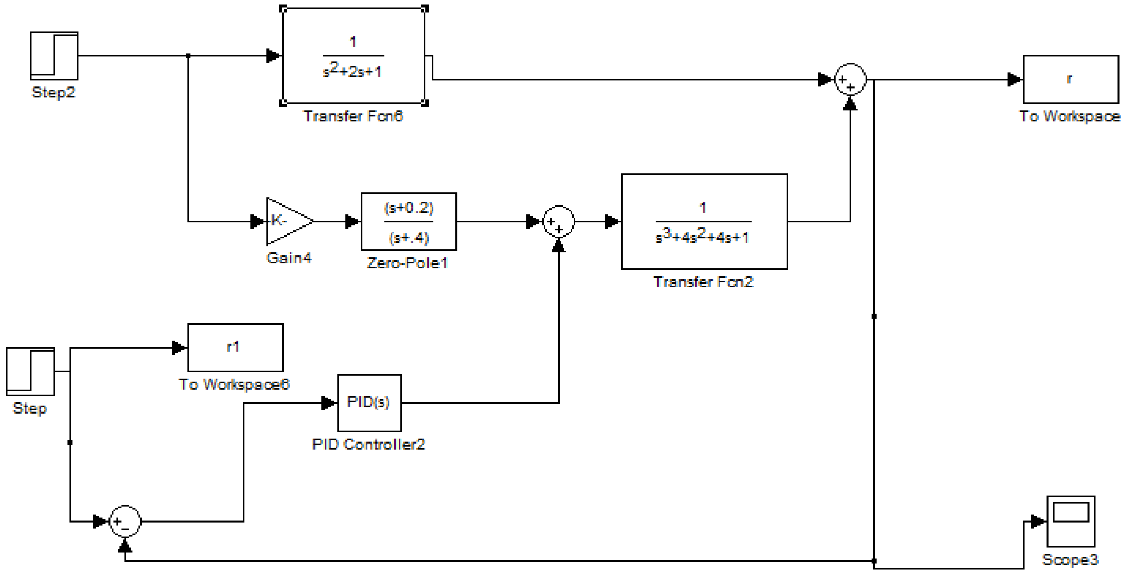

Fig. 5.14 Feedback and feedback-feed forward controller for three tank Interacting system (case1) with disturbance.

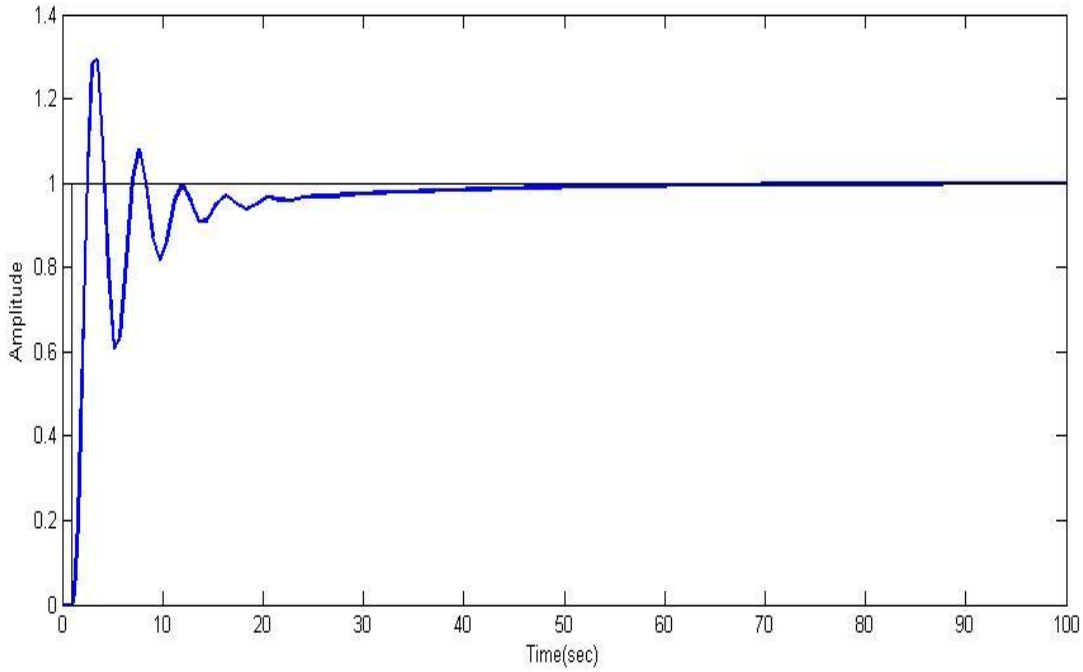

Fig. 5.15 Output response of feedback and feed forward-feedback controller for three tank Interacting system (case1) with disturbance.

5.2.3 Three tank interacting system (case2)-

Response of Three tank interacting system is shown below in which first tank and second tank are connected in non- Interacting with each other while second tank third tank are connected in Interaction with each other the transfer function between level of third tank and input flow to the first tank is calculated and then corresponding PI controller is designed then disturbance is applied to the second tank and transfer function between level of third tank and input disturbance is calculated and Feed forward controller is designed.

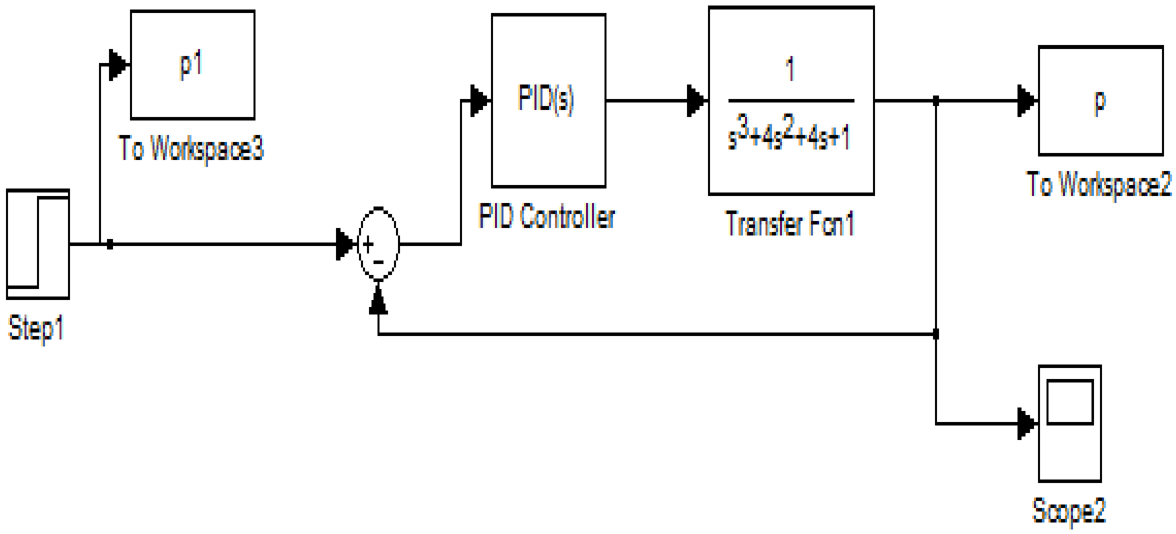

Fig. 5.16 Feedback controller for three tank Interacting system(case1) .

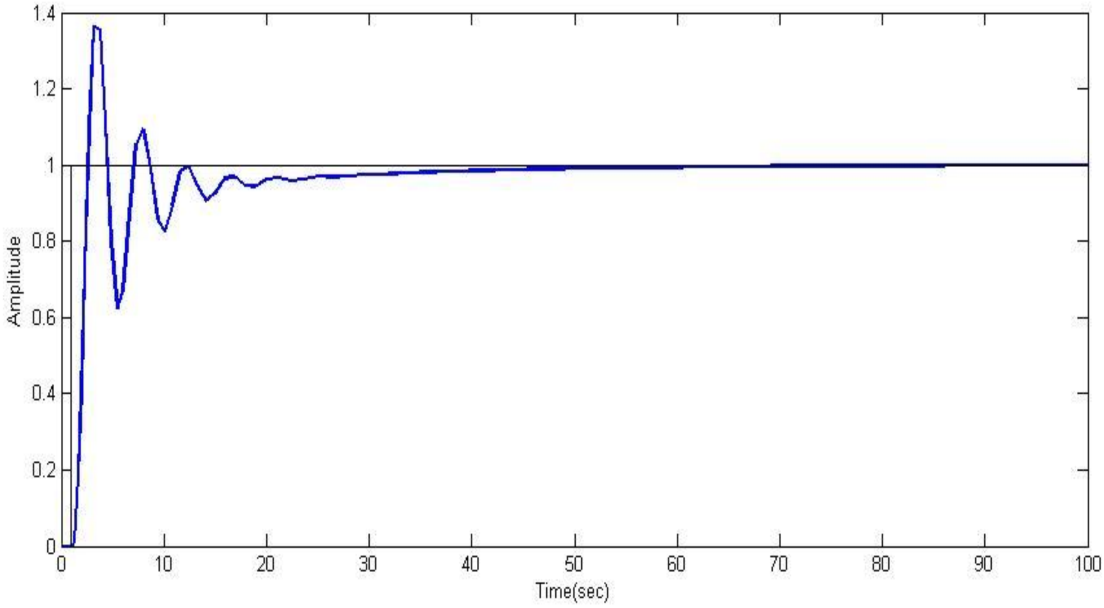

Fig. 5.17 Output response of feedback controller for three tank Interacting system (case2).

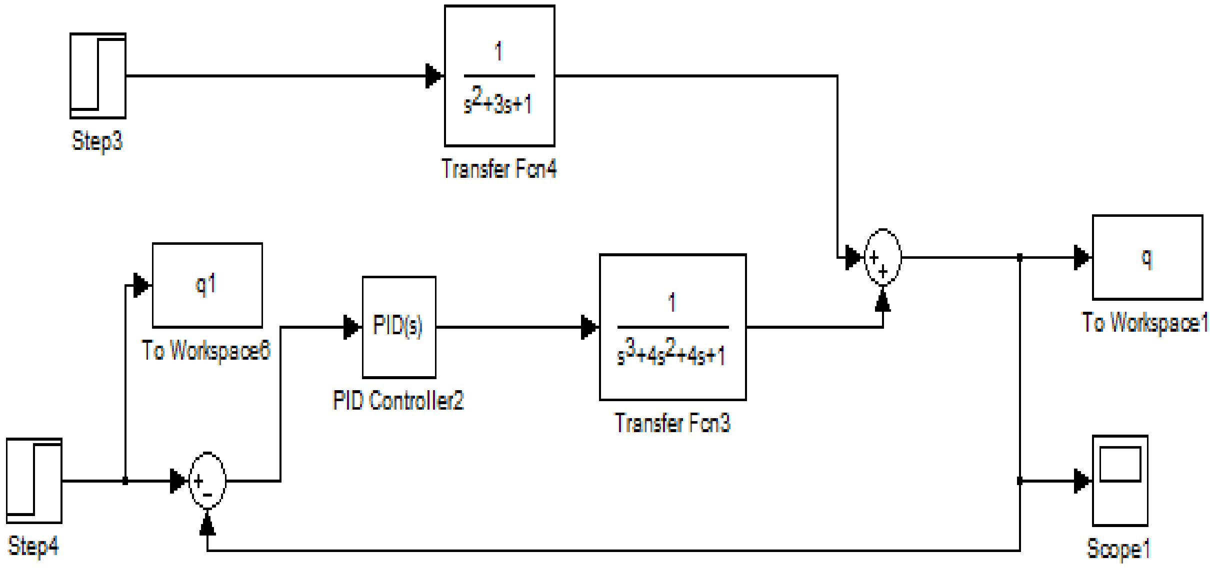

Fig.5.18 Feedback controller for three tank Interacting system (case2) with disturbance.

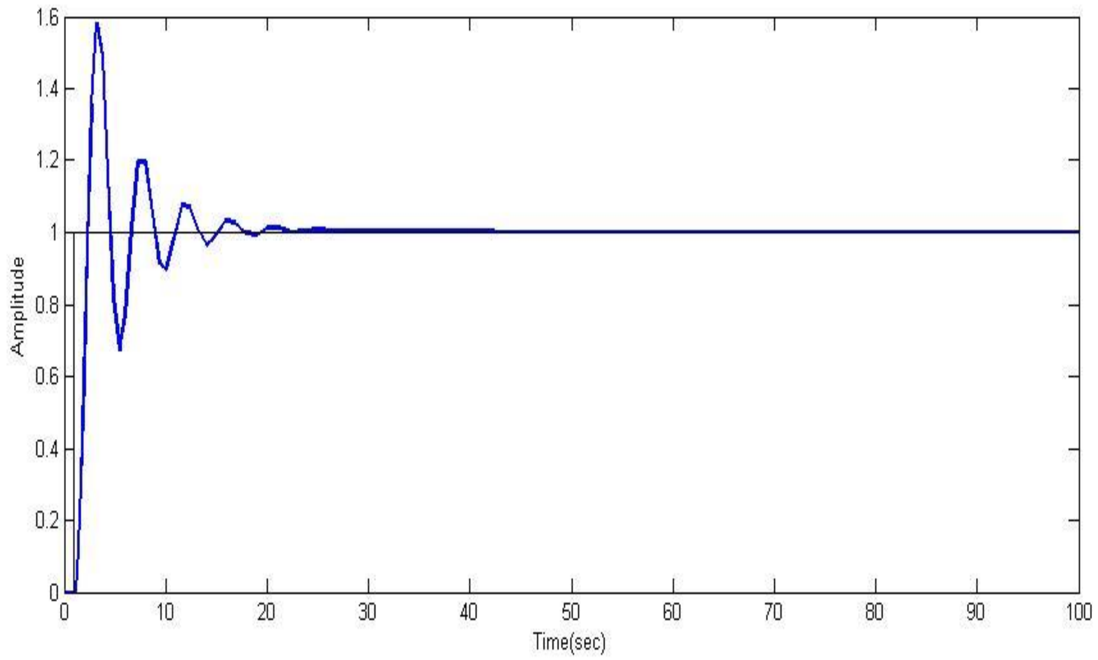

Fig. 5.19 Output response of feedback controller for three tank Interacting system (case2) with disturbance.

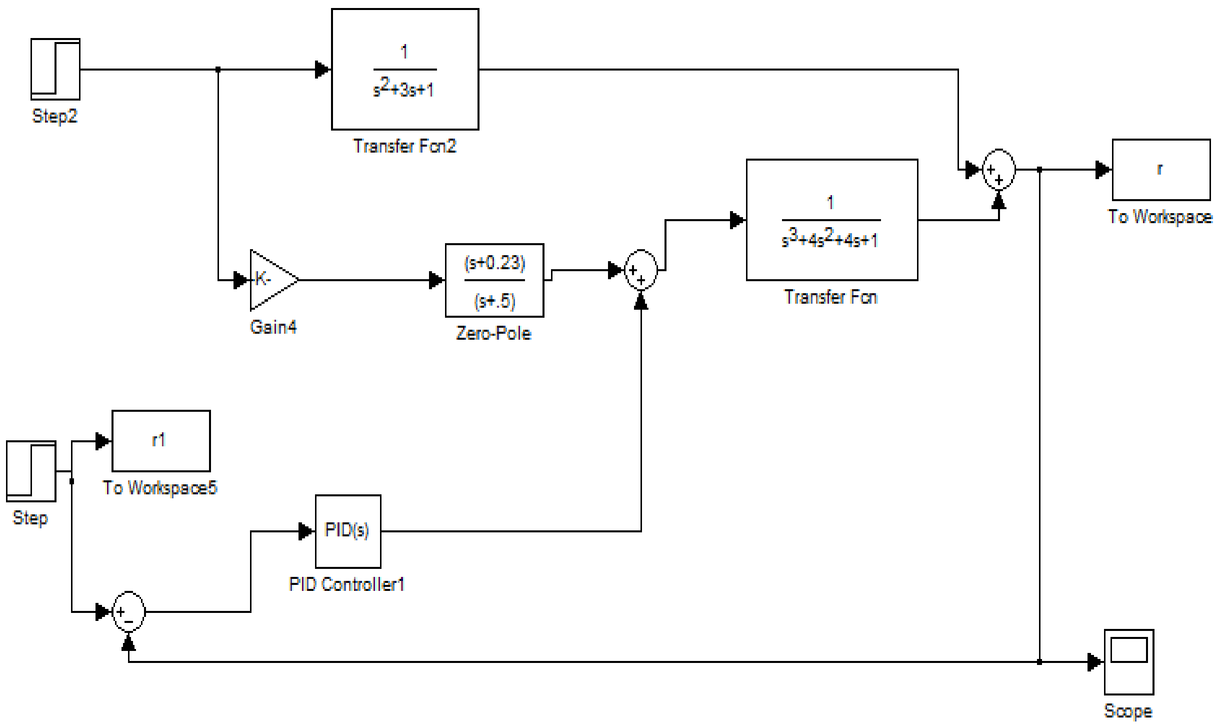

Fig.5.20 Feedback and feedback-feed forward controller for three tank Interacting system (case2) with disturbance.

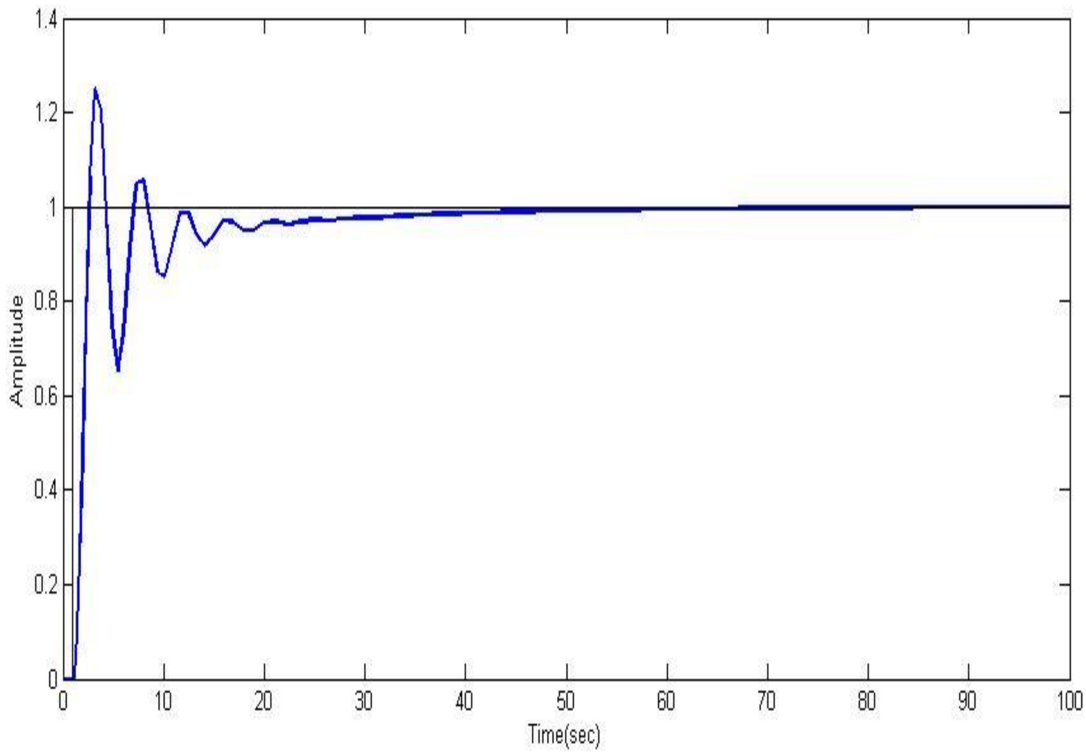

Fig.5.21 Output response of feedback and feed forward-feedback controller for three tank Interacting system (case2) with disturbance.

CHAPTER: 6

Conclusions and Future Scope

6.1 Conclusion

Response for various types of connections of the multi tank system which is connected in Interacting and non Interacting mode has been analyzed. Then the response is analyzed by designing various types of controller like feedback, feed forward and combination of feed and feed forward.

In interacting two tank system, first we calculate the relative gain array and then the decoupling circuit is designed and finally response is observed by designing a PI controller with step input.

For Interacting and Non Interacting system, first we observe the response without applying the disturbance to the system and then we are applying the disturbance to the second tank.

By applying the disturbance to the system peak overshoot of the system increased. To reduce the effect of these disturbances we are designing the feed forward controller. By using combination of feedback and feed forward- feedback controller, peak overshoot of the system is minimized.

6.2 FUTURE SCOPE

1. Feedback and combination of feed forward- feedback controller can be used for observing the response of four tank order tank.
2. Controller design for recycling application.
3. Decoupling design for three tank four tank system.

Bibliography

[1] Ravi V R, Thyagarajan T, "A De-centralized PID Controller for Interacting Non linear Systems," Proc. of IEEE international Conference on Emerging trends in Electrical and Computer Technology, pp. 297-302, 2011.

[2] Ravi V R, Thyagarajan T, "Application of Adaptive Control technique for Interacting Non linear Systems," Proc. of IEEE 3rd international conference on Electronics Computer Technology (ICECT), pp. 386- 392, 2011.

[3] Ravi V R, Thyagarajan T, "Gain scheduling Adaptive Model Predictive Controller for Conical Tank Interacting Level system" proc. of IEEE international conference on computing communication and network technologies, pp. 1-7, 2012.

[4] Parag Joshi, "Design of De-coupler for an interacting tanks system" IOSR Journal of Electrical and Electronics Engineering, vol.7, issue 4, pp. 48-53, 2013.

[5] Surekha Bhanot, "Process Control Principles and Applications", India: Oxford University Press, 2008.

[6] B, Wayne Bequette, "Process Control Modeling design and simulation", PHI Publication,2003.

[7] Donald R.Coughanower, "Process systems Analysis And Control",2nd ed.McGraw Hill international edition

[8] I.J. Nagrath., M. Gopal, "control system engineering", 5th ed. New age International Publishers,2010

[9] Gude. J.J, "Modified Ziegler-Nichols method for fractional PI controllers", international conference on Emerging Technologies and Factory Automation (ETFA), 2010, pp.1-5

Dissemination of the Research Work

[1] Ashish Kumar Singh and Sandeep kumar " Comparing the Performance Analysis of Three Tank System using Feedback and Feed forward- Feedback configuration", proc. of IEEE international conference on Advance Communication, control and computing technologies (ICACCCT), may- 2014, Tamilnadu, india.